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Extratropical low-frequency variability as a low-dimensional problem I: A simplified model

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SUMMARY

It is shown that the low-frequency and large-scale variability of an intermediate complexity reference model can be reproduced faithfully by a simplified model of 10 independent variables and 10 equations. The reference model is quasi-geostrophic and baroclinic. The low-order model is based on the truncated projection of the reference-model equations on the empirical orthogonal functions of its output. A closure term is shown to be essential for good performance of the low-order model. This closure term is meant to reproduce all the neglected scales and all the scale interactions, mainly baroclinic eddy forcing, that drive the large-scale flow. The closure is built as an empirically defined function of the large-scale flow of the model, relying on an extensive previously computed library of tendency differences between the full and truncated model. Two other parametrization schemes, a time-mean and a stochastic one, are tested; comparisons of these two with the former underline the importance of the flow dependence of the formulation. The low-order and reference models exhibit the same climate, as well as the same low-frequency variability and weather regimes.

KEYWORDS: Extra-tropical circulation Low-frequency variability Low-order modelling

1. INTRODUCTION

The history of the scientific works dedicated to understanding the dynamical mechanisms at the base of atmospheric low-frequency variability is now rather long. In the late forties and early fifties (Baur 1947; Rex 1950) the first studies suggested the existence of long-lived circulation anomalies (typically atmospheric blocking) in the extratropics, that were difficult to explain by linear planetary wave theory. In the early eighties these phenomena were addressed again, following the work of Charney and Devore (1979), who showed that the nonlinearity of the quasi-geostrophic (QG) equations could lead to multiple equilibria in the extratropical atmosphere.

Rather than being described as a superposition of linear planetary Rossby waves, the large-scale extratropical atmospheric dynamics was then thought of as a succession of a few 'weather regimes', whose patterns were classified using various statistical techniques such as cluster analysis (Mo and Ghil 1986; Molteni *et al.* 1990; Kimoto and Ghil 1993a,b; Cheng and Wallace 1993; Michelangeli *et al.* 1995; see also Smyth *et al.* 1999 for a review), or found as equilibria or quasi-equilibria of the large-scale atmosphere (Legras and Ghil 1985; Vautard 1990; Haines and Hannachi 1995; Itoh and Kimoto 1999). The signature of multiple weather regimes has also been found by studying the probability density function (PDF) of the atmospheric state projected onto a one-dimensional phase space (Hansen and Sutera 1986).

Two principal mechanisms of maintenance of these weather regimes have been proposed so far. On the one hand, along the lines of Charney and Devore (1979), Reinhold and Pierrehumbert (1982) and Legras and Ghil (1985) suggested that the regimes are associated with stationary solutions of the governing equations which are, however, unstable; hence the atmospheric state cannot 'freeze' into a regime without being transported to another one after some time. The existence of multiple stationary solutions is tightly linked with the nonlinear resonance of some Rossby waves with the topography.

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On the other hand, Shutts (1983) found in a simplified channel model without topography that the blocking 'modon' can be maintained by the sole feedback of transient waves, artificially produced, which could for instance represent the strong baroclinic activity present over the western margins of the two main oceans. Vautard and Legras (1988) showed that these baroclinic transients could indeed maintain several weather regimes, but these cannot be obtained directly as stationary solutions without taking into account the mean transient feedback they generate.

In any case, these studies suggest that an important part of the low-frequency variability takes place in a phase space of much smaller dimensionality than the dimension of the set of equations generally used to simulate it. This low-dimensional character is evident when decomposing the low-frequency variability into its empirical orthogonal functions (EOFs): the major part of the variance can be described by a reduced number (typically 10–50) of EOFs. More quantitative measures of this dimensionality have been carried out agreeing with this range of values (Lorenz 1969; Wallace *et al.* 1991 or Fraederich *et al.* 1995).

Starting from the hypothesis that atmospheric low-frequency variability is a low-dimensional problem, it should be possible to formulate a reduced set of equations describing its evolution in a relatively realistic way. However, one faces two major difficulties. The first is linked with the choice of the variables to be used, and the second deals with the closure problem, i.e. the feedback of unresolved variables.

In order for the model to produce realistic patterns, the low-order phase space must include a set of basis functions that describes a large fraction of the variance, hence good candidates are the leading EOFs. Several attempts to formulate EOF-based models have been carried out, normally with the aim of eventually reducing the numerical cost of short-term forecast models. For instance, Rinne and Karhila (1975), Schubert (1985) and Selten (1993, 1995) could successfully reduce the dimensionality of simple barotropic models by projecting the equations onto a limited set of EOFs, without changing the dynamics significantly. However, attempts to reduce baroclinic models proved to be somewhat less successful (Selten 1997). Recently, though, EOF modelling has started to be applied to simplified climate modelling; an example is found in Achatz and Branstator (1999). The present work is very much in the same rationale.

The problem of closure is to correctly represent the effect of unresolved variables. If the prognostic variables include only large-scale, equivalent barotropic patterns, one must parametrize in particular the effect of the baroclinic transients, which is known to be crucial to the maintenance of the low-frequency variability (Egger 1981; Metz 1986; Hoskins 1983). As shown by many authors, the transient feedback is generally not dissipative and therefore cannot be represented by some kind of hyperviscosity.

In an attempt to deal with the above two difficulties, Da Costa and Vautard (1997) built a deterministic model, based on only a small number of EOFs of the potential vorticity on an isentropic surface. In this model, the effects of the unresolved scales (in particular the baroclinic transients) are fully parametrized by an empirical closure using analogues of the flow. In fact, in this model, the full tendency of the potential vorticity was modelled using analogues. The shortcoming of such an approach is that the equations of their reduced model are not known. Moreover, the model was restricted to the North Atlantic sector.

In this paper, we wish to demonstrate that the northern hemisphere (NH) extra-tropical low-frequency variability has a low dimensionality. A low-order (10 variables) dynamical model of the flow is built, based on the projection of a reference QG baroclinic model (Marshall and Molteni 1993) onto a subspace spanned by its leading EOFs. In other words, an approach is chosen similar to the 'perfect model' one, that is often used in predictability studies. The equations of the model are projected directly onto the EOF subspace, and only the closure is achieved by an empirical method. Such empirical closure has been constructed employing a technique similar to that previously used by D'Andrea and Vautard (2000, hereafter DV2000) but with different motivations. Using this low-order model, we only attempt to simulate the part of the atmospheric variability that is driven by internal processes, and not external forcing such as ocean–atmosphere exchanges, hence the choice of the QG model as the reference to be simulated. We show that the QG-model atmosphere can be faithfully reproduced by the low-order model, hence demonstrating the low-dimensional behaviour. The skill of the low-order model is measured by its ability to reproduce the climatology and the weather regimes of the reference model.

In section 2 the reference model is described briefly. Section 3 describes the methodology used to build the low-order model. The choice of an EOF orthogonal basis and the choice of the closure scheme are addressed. In section 4 it will be shown that the low-order model reproduces very well the slowly varying part of the reference model. The basic climatologies of the low-order model with different formulations of the closure will be analysed and compared with the reference model. The importance of transient eddy feedback in the dynamics of the truncated model will become apparent. Section 5 summarizes and concludes, further discussing the methodology and giving some future applications of the low-order model.

2. The reference model

The reference model is that built by Marshall and Molteni (1993, hereafter MM93) to which the reader is referred for further details. The model is spectral on the sphere, with a triangular truncation at total wave number 21. Vertical discretization is performed in pressure coordinates at three levels (200, 500 and 800 hPa). On each level, the QG potential-vorticity equation with dissipation and forcing is integrated:

$$\frac{\partial q}{\partial t} = -\mathrm{J}(\psi, q) - D(\psi) + S \tag{1}$$

where J represents the nonlinear Jacobian operator and ψ is the QG stream function, linked to the QG potential vorticity q by a relation $q = \mathcal{L}\psi$. The linear dissipation term D includes in a simple way the Ekman dissipation (orography dependent) and a Newtonian relaxation between the layers. The orographic contribution to the potential vorticity at the lower layer is included in the linear differential operator $\mathcal{L}$. The forcing term S is designed to include the sources of potential vorticity that result from processes not explicitly included in the equations. These phenomena are typically sea–atmosphere interactions, diabatic heat fluxes (linked for example to precipitation etc.) and the effect of the divergent flow. On top of this the forcing implicitly contains the effects of subgrid-scale processes. The forcing term has been estimated empirically by MM93 as follows.

Having a long series of analysed states, $\widehat{q}$ and $\widehat{\psi}$, one can write, by averaging (1) in time,

$$\overline{S} = \overline{\mathrm{J}(\widehat{\psi}, \widehat{q})} + D(\overline{\widehat{\psi}}). \tag{2}$$

With this mean-source term, computed from a long dataset of observed winter data, MM93 (and many other authors thereafter) performed long integrations of the model, which showed a satisfactory degree of realism. Equation (2) can, in turn, be written as

$$\overline{S} = \mathrm{J}(\overline{\widehat{\psi}}, \overline{\widehat{q}}) + D(\overline{\widehat{\psi}}) + \overline{\mathrm{J}(\widehat{\psi}', \widehat{q}')}, \tag{3}$$

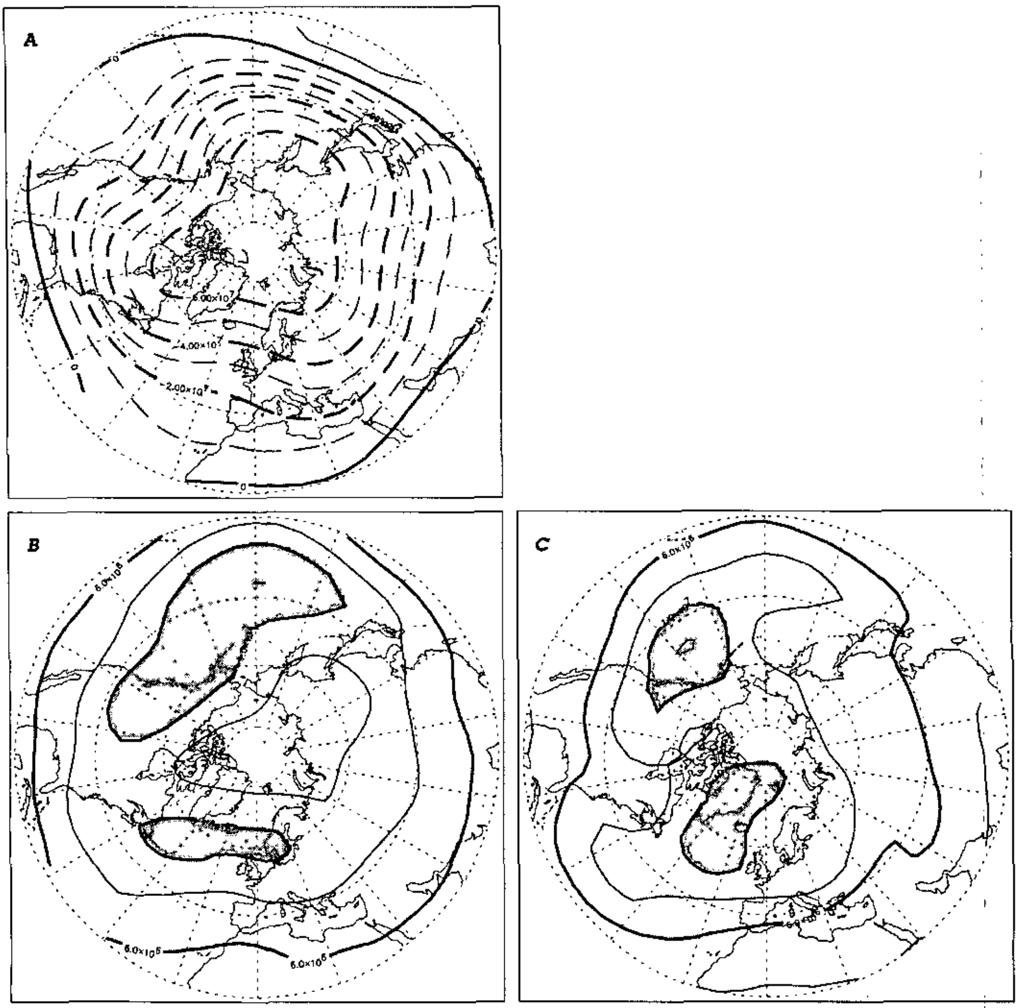

Figure 1. 500 hPa stream function climatology of the integration of the reference quasi-geostrophic model. (a) Mean field, contours every 10^7 m^2s^{-1}, negative contours dashed; (b) high-frequency standard deviation, contours every 2×10^6 m^2s^{-1}, with shading over 10^7 m^2s^{-1}, (c) low-frequency standard deviation, contours and shading as (b).

where overbars denote time averages, and primes deviations from these. The latter decomposition allows the first two terms of the right-hand side (r.h.s.) of (3) to be interpreted as imposing a basic state, and the last term as adding the average contribution from transient eddies to the flow. A necessary condition for the model to reproduce the exact climatology is that its transient fluxes exactly match the contribution of $\overline{J(\widehat{\psi}', \widehat{q}')}$.

The only difference between the model used by MM93 and the present one is the numerical time integration scheme; here a predictor–corrector (first order Adams–Bashforth–Mouton) is used, while MM93 used a leap-frog scheme. The time step used is one hour. A climatology of 500 hPa stream functions of a long (10 000 days) integration of the model is shown in Fig. 1; it includes the long-term mean, and high- and low-frequency standard deviations. Hereafter this integration will be referred to as the *reference simulation*. The forcing term used in the reference integration was calculated by (2) using a twice-daily European Centre for Medium-Range Weather Forecasts (ECMWF) analysis dataset ranging from December 1984 to February 1994 for December–January–February; consequently, the integration can be considered as a perpetual winter one. Only NH middle and high latitudes are considered, for the QG approximation is only valid in the winter extratropics, and S is computed from NH winter data. The southern hemisphere (SH) climatology, with this forcing term, is in general less realistic, and the model is integrated globally basically to avoid boundary condition problems.

The reference model has a climatology that reproduces quite realistically the long-term observed one (Fig. 1). Low- and high-frequency variabilities are computed by a simple 10-day square filter (the standard deviation of 10-day means and of the departures from 10-day means). The model has a Pacific and an Atlantic storm track, and a quite realistic low-frequency activity maxima at the end of the storm tracks. See D'Andrea and Vautard (2000) for a further comparison of the reference-model climatology and the observations.

3. THE LOW-ORDER MODEL

(a) *Defining a basis*

The problem to be solved here, is to build a model containing only a few variables but which still simulates realistically the low-frequency variability of the reference QG model. One approach could be based on simply performing a severe truncation (T5 for instance) in the spherical-harmonics space. The main drawback is that the subspace spanned by these basis functions does not necessarily contain the most interesting part of the variability. In other words, the 'low-frequency attractor' may simply not be contained within the space spanned by the leading spherical harmonics.

Instead, the approach of Selten (1995) is followed here, and the basis formed by the leading EOFs of the reference model is used. The EOF decomposition requires the choice of a norm. As in Selten (1995), the square stream function norm is chosen. More physically relevant norms, involving potential enstrophy or total energy, that are based on constants of motion of the linearized or of the non-dissipative cases, probably deserve a specific study that is beyond the scope of this article. EOFs are computed from the output of the reference integration of the QG model. In practice, the EOF calculation is performed on the spherical-harmonic coefficient space, considering the three vertical levels simultaneously. Hence each EOF is a three-level stream function field. The total number of non-zero coefficients in this model is 1449, so 1449 EOFs are calculated and sorted by decreasing order of their associated eigenvalues.

Once the basis has been estimated, the model equations (1) are projected onto their spanning space. The advection term can be expressed as a bilinear form:

$$A_{ijk} = \langle -\mathrm{J}(\mathbf{e}_i, \mathcal{L}\mathbf{e}_j), \mathbf{e}_k \rangle, \tag{4}$$

and the dissipation term as a matrix:

$$D_{ik} = \langle D(\mathbf{e}_i), \mathbf{e}_k \rangle, \tag{5}$$

where $\mathbf{e}_i$ represents the ith basis vector and $\langle \cdot, \cdot \rangle$ is the scalar product associated with the chosen norm. Once the operators A and D are defined, the model can be integrated. For example the evolution of the kth component of the stream function will be as follows ($0 < k \leq K$, where K is the truncation):

$$\frac{\partial \psi_k'}{\partial t} = \sum_{i,j=1}^{K} A_{ijk}\, \psi_i' \psi_j' + \sum_{i=1}^{K} D_{ik}\, \psi_i' + S'. \tag{6}$$

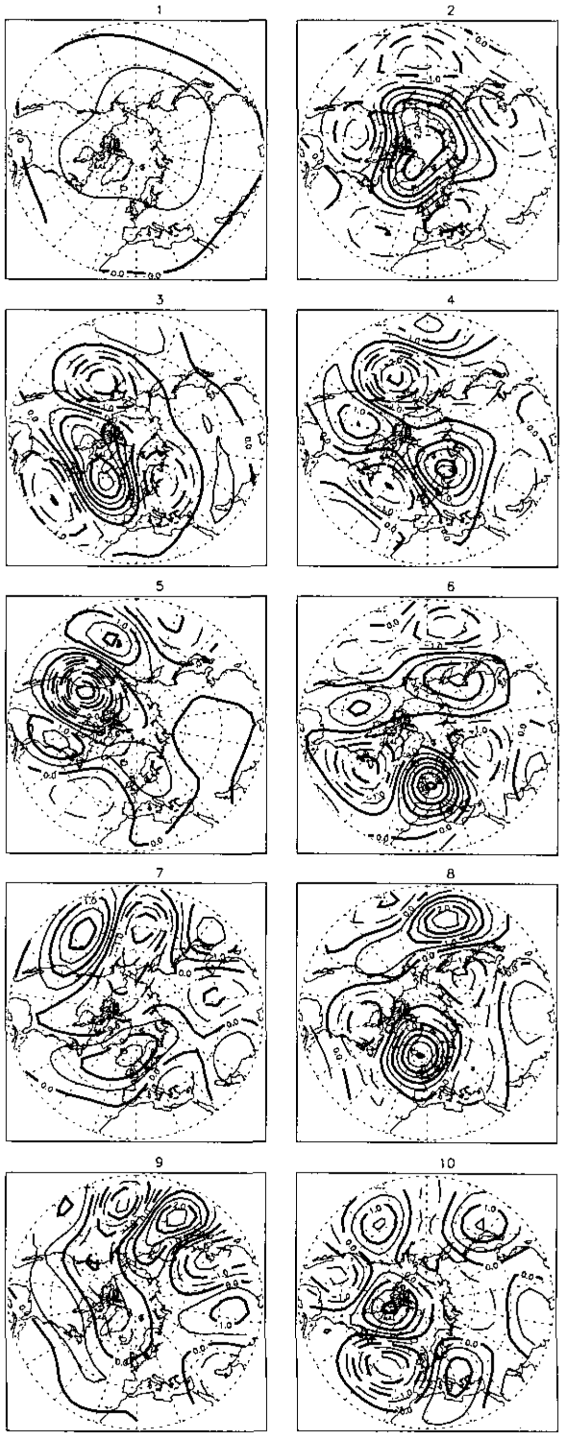

Figure 2. The first ten vectors of the empirical orthogonal function base over the northern hemisphere 500 hPa geopotential height field. Contours are every 0.5, with negative contours dashed.

S' stands for the closure, or forcing, that is described in the next subsection. The primes are used to remind that the vectors ψ and S are expressed on the EOF base. The truncation chosen is $K = 10$, which represents around 37% of the total variance. This choice is rather arbitrary, but similar results to those shown below have been found with 15 and 25 EOFs.

The NH part of the first 10 EOFs, or base vectors, is shown in Fig. 2; only the 500 hPa level is shown. The vectors have an equivalent-barotropic structure. It should be noted that the EOFs were computed without previously subtracting the long-term mean from the fields. Consequently the first base vector is very close to the climatology. Selten (1995) found that this approach does not give different results from subtracting the mean and constructing an EOF model for the anomalies.

Some of the EOFs offer a straightforward interpretation. EOF 2 has a wave number 4–5 aspect, together with a quite zonally symmetric structure, indicating an oscillation of the meridional stream function gradient. Over the Atlantic ocean it has, nevertheless, a projection onto the North Atlantic Oscillation (NAO) structure of Wallace and Gutzler (1981). EOF 3 looks more like a Pacific North American (PNA) pattern accompanied by another NAO-like pattern over the Atlantic. Higher basis vectors exhibit Rossby wave-like structures (e.g. EOF 7 and 9), or other known features, for example EOF 6 and especially EOF 8 carry the signature of atmospheric blocking in the Euro-Atlantic sector.

(*b*) *Model closure*

In the reference model, the forcing term is supposed to represent all the processes and phenomena that are not included in the equations, as discussed in section 2; these include diabatic heat fluxes, sea–air interaction etc. In the case of the EOF-truncated model, the unresolved phenomena include additional adiabatic processes such as nonlinear interactions between large and small scales. Among these interactions is the feedback of transient baroclinic waves that are induced by the instability of the large-scale flow.

In order to include these new effects, three different types of forcing parametrizations have been used in the present study: a 'constant-forcing' closure, a 'flow-dependent' closure, and a 'stochastic' closure.

The constant-forcing closure is obtained by applying directly the MM93 procedure (2) for the low-order model, but starting from reference-model states instead of the ECMWF analysis. In this way, the model forcing includes the mean effects of the phenomena that are eliminated by the truncation, but does not take into account the variable part of this forcing, due for instance to the change of the large-scale flow pattern and its subsequent storm track.

For the flow-dependent closure, almost the same technique as in DV2000 is used. In another framework, DV2000 developed a method of eddy forcing parametrization as a form of model error correction. This parametrization was applied, incidentally, to the same model used here as reference. They introduced a flow-dependent forcing term in the equations in order to improve substantially the model climatology. This forcing was calculated in DV2000 as the average, over analogues of the large-scale flow, of the difference between the instantaneous tendency and the model tendency calculated from the equations (including advection and dissipation).

In the same line of thought, first of all a library of tendency errors, or residuals, was constructed for the truncated model with respect to the reference model. That is, for all times t:

$$R_t = \left.\frac{\partial \psi_t}{\partial t}\right|_{\text{ref}} - \left.\frac{\partial \psi_t}{\partial t}\right|_{\text{trunc}}, \tag{7}$$

where ψ_t is the state of the reference integration at time t. This library is the time series of the 'ideal' correction terms that have to be added to the truncated equations for them to equal the reference ones. The information included in this time series can be used to construct a forcing term for the truncated model that gives a correction for any value of ψ.

Once the library of tendency errors is constructed, the following steps are followed in order to construct a time-dependent forcing term.

(i) At every time step the distance of the model state $\psi'(t)$ from each reference-model field is computed. The distance is defined as the Euclidean distance in the space of the first K EOFs.

(ii) The reference-model fields are sorted according to the distance from $\psi'(t)$.

(iii) With R_i the residual corresponding to the ith closest reference-model state, the forcing term at time t is computed as follows:

$$S(t) = \frac{\sum_{r_i < r_0} R_i\{1 + \cos(\pi \frac{r_i}{r_0})\}}{\sum_{r_i < r_0}\{1 + \cos(\pi \frac{r_i}{r_0})\}}, \tag{8}$$

where r_i is the distance of the ith closest reference-model state and r_0 is a chosen cut-off distance.

The definition in (iii) is nothing more than a weighted mean of analogues with a sinusoidal weight. In DV2000 this weighting function was uniform and the analogues were simply averaged. Here weighted averages are used in order to make the forcing term differentiable with respect to r. The importance of the differentiability of $S(\psi)$ will be made clear in Part II.

The cut-off distance r_0 controls the variance of the source term. The higher the value of r_0 the more analogues will be found, the more R's will be averaged and the smaller the variance of $S(\psi)$ will be. Since the correct variance to be given to the forcing term is unknown, the only way to tune it is to choose the r_0 that gives, a posteriori, a correct model response on the output variance. See the discussion in DV2000 on this point. In this paper $r_0 = 6.3 \times 10^6$ was chosen; this corresponds to the third percentile of the distribution of all distances of the reference-model states.

During the low-order model integration, it may occasionally happen that the model trajectory exits the area of reference-model states, and no analogues can be found closer than r_0. In this case the closest five analogues are used. This happened only four times during a 10 000 day integration, and the trajectory always went back to 'inhabited' areas within a few days. In DaCosta and Vautard (1997), to solve a similar problem, a linear relaxation towards climatology was used instead.

Finally, a 'stochastic closure' was devised, basically for comparison with the flow-dependent closure. In this case the forcing term is also time-dependent, but the variation is purely stochastic and does not take into consideration its dependence on the large-scale flow. The stochastic-forcing term was constructed as follows: starting from the time series of the forcing terms obtained in the flow-dependent case, another time series is built by a regressive model of order one having the same lag-0 and lag-1 covariance matrices. The model is then integrated and forced at every time step by successive terms of the new time series.

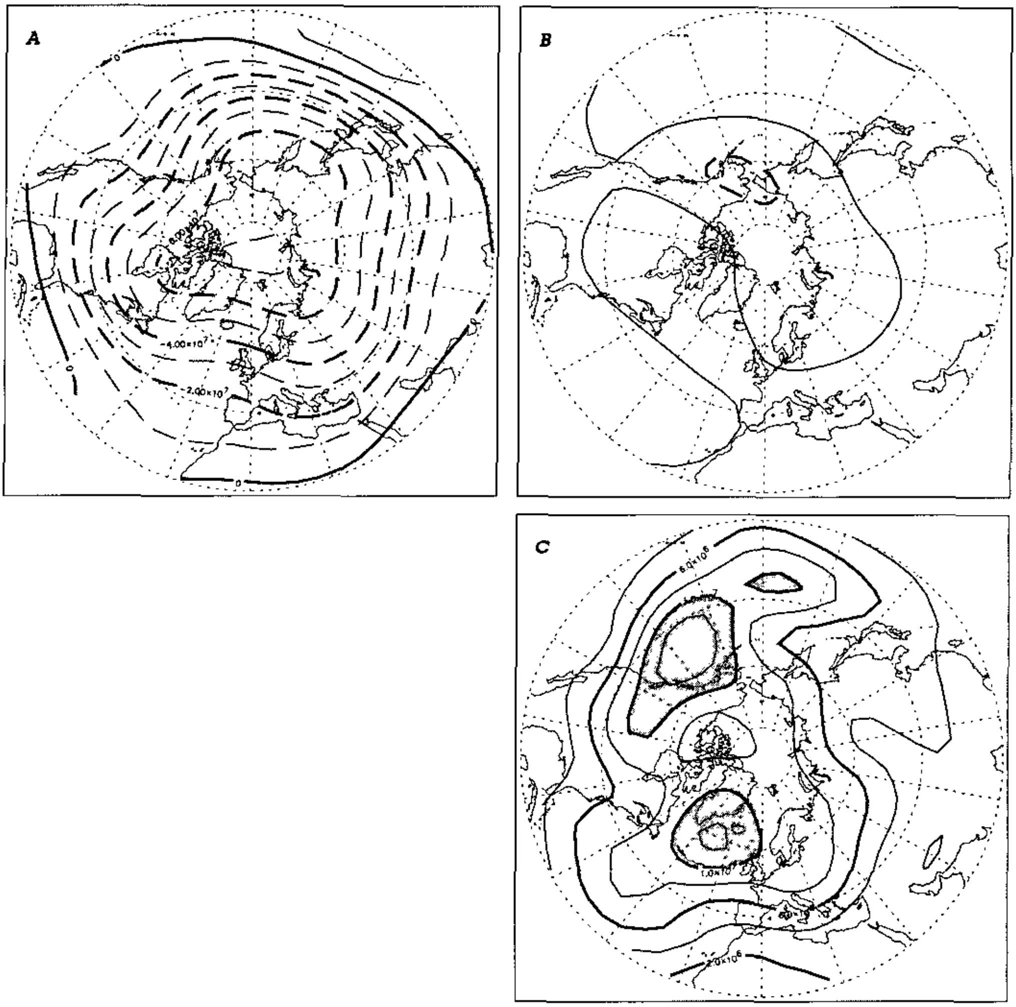

Figure 3. 500 hPa stream function climatology of the low-order model integration with flow-dependent forcing. (a) And (c) as in Fig. 1. (b) The difference between low-order model climatology and reference-model climatology, contours every 2×10^6 m^{2}s^{-1}.

4. LOW-ORDER MODEL CLIMATOLOGY

The scope of this section is to investigate to what extent the low-order model, with the three above closures, reproduces the climate of the reference model. The comparison between the different formulations of the closure allows an assessment of the best way to parametrize the truncated scales (mainly transient eddies) in the resolved part of the model. The comparison between flow-dependent and constant forcing tells whether the mean effect of the transients is sufficient to maintain the low-frequency variability. The comparison of the flow-dependent and stochastic forcing tells whether the flow dependency of the transients must be taken into consideration in order to correctly represent their interaction with the resolved flow, or if a random variability added on top of the mean term is sufficient.

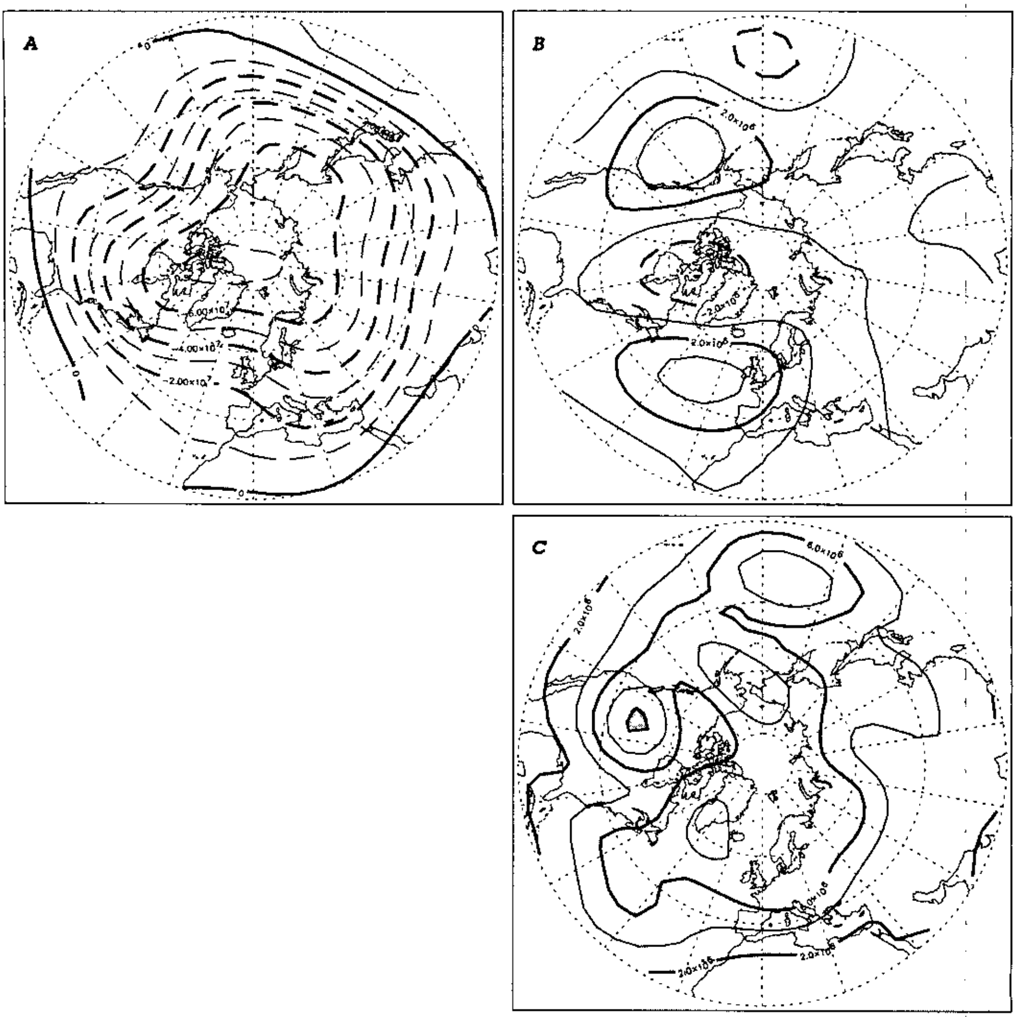

Figure 4. As Fig. 3 but for the constant-forcing model.

(*a*) *Mean state*

In Fig. 3, a 500 hPa climatology for the low-order model with flow-dependent closure is shown, analogous to that of Fig. 1 for the reference model. The climatology is obtained from an integration of 10 000 days. Only the low-frequency variability is shown; in the case of the low-order models, that are not designed to simulate the high-frequency variability, almost all the variability takes place at time-scales lower than 10 days and the high-frequency variability map is uninteresting. In contrast to Fig. 1, Fig. 3(b) shows the difference of climatology between the low-order and reference models (the so-called systematic error).

It can be seen that the difference in climatology between the two models is extremely small (compare Fig. 1(a) with Fig. 3(a) and consider Fig. 3(b)). In terms of geopotential height, the maximum systematic error corresponds to only about 27 m, over the Bering Strait.

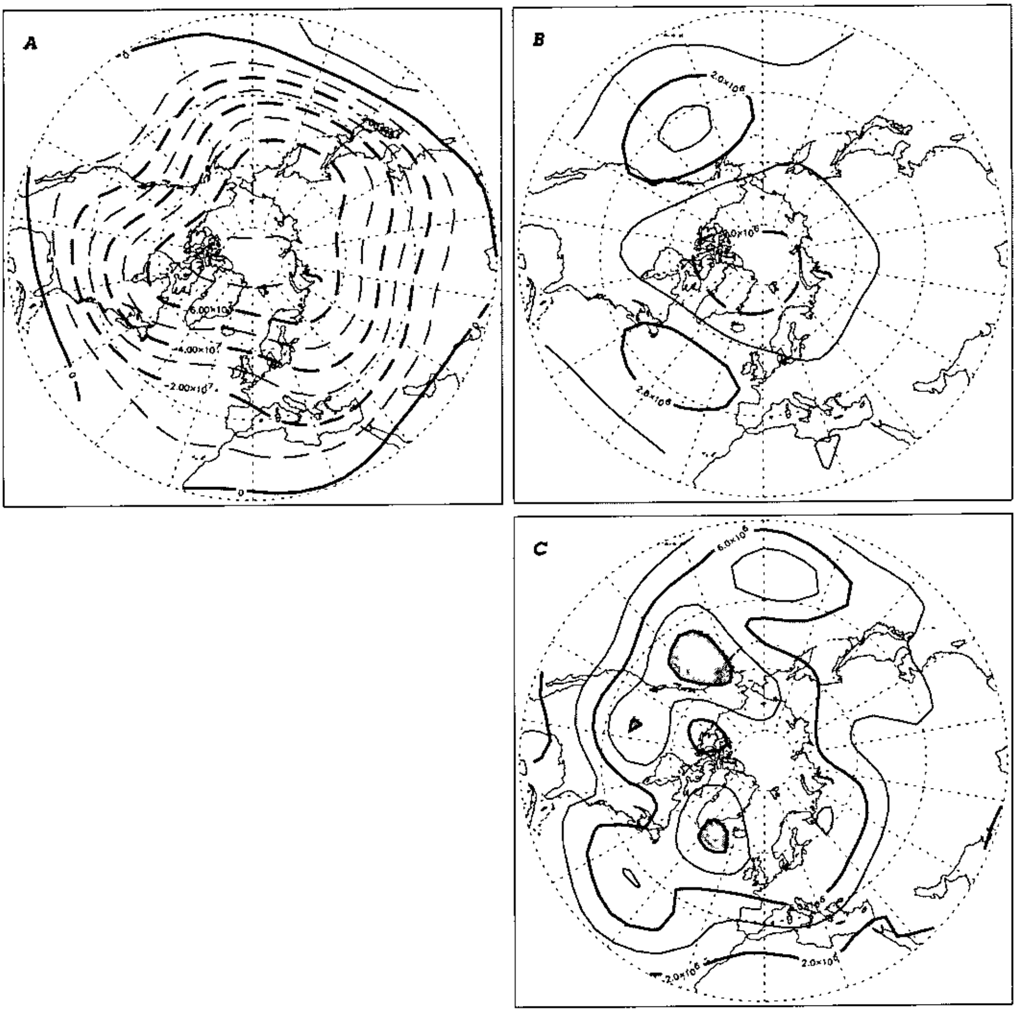

Figure 5. As Fig. 3 but for the stochastic-forcing model.

The low-frequency variability is also realistically reproduced, with the two maxima well located over the Atlantic and Pacific oceans, and with approximately the same amplitude as in the reference model.

The climatology maps for the constant-forcing and the stochastic-forcing cases are shown in Fig. 4 and Fig. 5.

The constant-forcing model (Fig. 4) shows a larger systematic error, with geopotential height anomalies up to 55 m. The variability is also much underestimated; it increases in the stochastic-forcing case (Fig. 5), and has almost the same peak amplitude as in the reference model (compare with Fig. 1), nevertheless the systematic error still presents larger anomalies than in the flow-dependent case.

Consequently it can be seen that: on the one hand, the mean formulation of the transient eddies effect is not sufficient, the model clearly has not enough variability and must be given more energy by a variable forcing term; on the other hand, the stochastic formulation of variable forcing is quite able to increase the variability, and

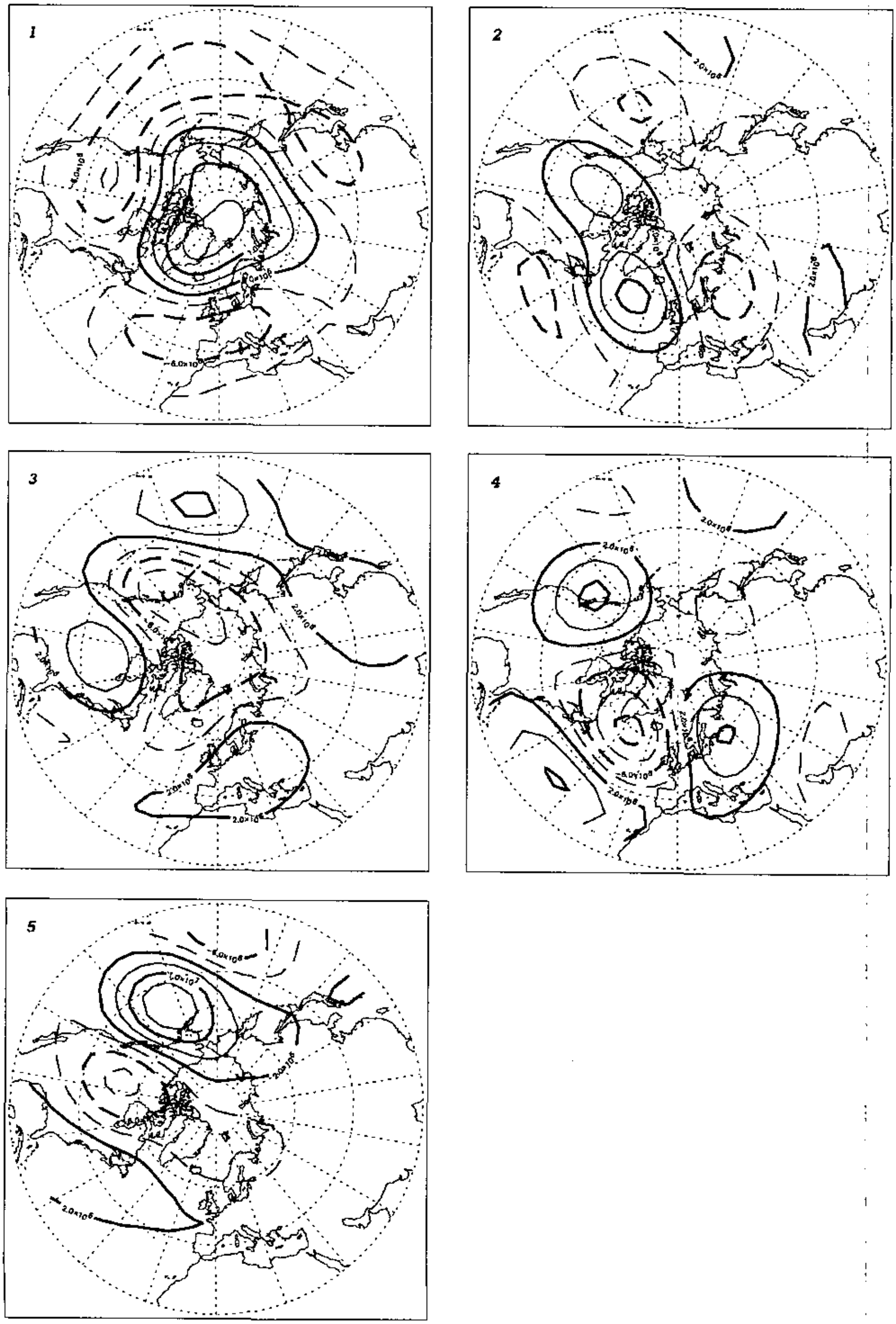

Figure 6. Hemispheric cluster centroids of the reference model at 500 hPa. Classification in five classes. Contours are every 4×10^6 m^2s^{-1}, with negative contours dashed.

TABLE 1. ANOMALY CORRELATION COEFFICIENTS OF THE REGIMES OF THE LOW-ORDER MODELS WITH THE CORRESPONDING REGIME OF THE REFERENCE MODEL FOR NUMBER OF CLASSES, k, BETWEEN 3 AND 6

Flow-dependent forcing						
$k=3$	0.95	0.97	0.93			
$k=4$	0.87	0.93	0.92	0.69		
$k=5$	0.96	0.71	0.98	0.97	0.77	
$k=6$	0.88	0.97	0.85	0.91	0.36	0.68
Constant-forcing model						
$k=3$	0.91	0.84	0.27			
$k=4$	0.19	0.87	0.94	0.81		
$k=5$	0.88	0.73	0.81	0.62	0.41	
$k=6$	0.75	0.88	0.30	0.85	0.62	0.49
Stochastic-forcing model						
$k=3$	0.93	0.91	0.79			
$k=4$	0.64	0.88	0.76	0.50		
$k=5$	0.81	0.59	0.85	0.82	0.28	
$k=6$	0.72	0.86	0.26	0.87	0.48	0.65

also somewhat reduces the systematic error, but less accurately than the flow-dependent parametrization.

These results confirm the findings of DV2000: the flow dependency of the transient forcing is essential for systematic-error reduction, although results here are from a much simplified model.

(b) *Weather regimes of the low-order model*

The low-frequency variabilities of the reference- and low-order models can be further investigated by comparing their weather regimes. First, the regimes are calculated for the reference model, in NH, by means of cluster analysis. We use the non-hierarchical clustering algorithm already proposed by Michelangeli *et al.* (1995). Given a number of clusters, this algorithm finds a partition of the data sample by minimizing the within-cluster variance. The reader is referred to Michelangeli *et al.* (1995) for the details of the method.

The data were first projected onto the first five EOFs of NH before performing the cluster analysis. Similar results were obtained with six and seven EOFs.

In Fig. 6 the reference-model regimes are shown as an example for five clusters. This number of clusters has been chosen by trial and error, and the reason for this choice will be made clearer below; here, weather regimes will be used as a diagnostic comparison tool. The cluster centroids are reminiscent of known patterns of variability of NH. Regimes 2 and 4 have a projection on the two phases of the NAO; also regime 1 recalls a negative phase of NAO in the Atlantic sector, but its anomaly is extended to the whole Arctic and is quite zonally symmetric; it represents a weakening of the zonal flow. Regime 5 rather resembles a PNA pattern.

For comparison between the reference model and the three low-order model versions, the stream function maps have been classified for a number k of classes ranging from three to six. The centroids of the regimes of the low-order models are then compared with the corresponding regimes in the reference model using anomaly correlation coefficients (ACCs). The results are shown in Table 1, which lists all the 'scores' of the different models for the different choices of k.

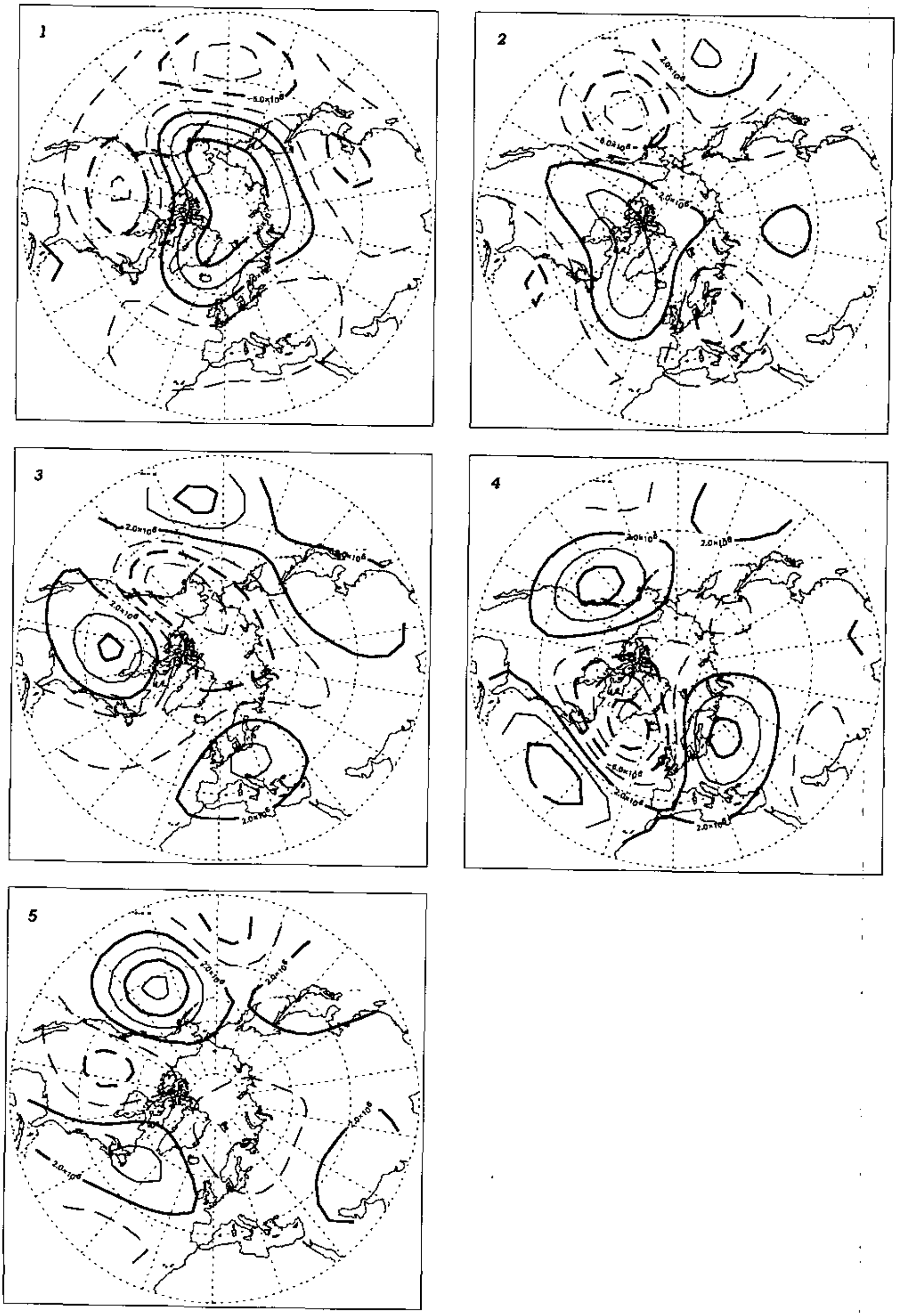

Figure 7. As Fig. 6 but for the low-order model with flow-dependent forcing.

From Table 1, it is evident that the flow-dependent forcing version is superior to the others. If one chooses a (severe) threshold of 0.7 of ACC to accept or discard the identification of two regimes, it can be seen that there is a good correspondence between the regimes of the reference model and of the flow-dependent model in a classification up to five classes (with the marginal exception the value of 0.69 at $k = 4$). This is the reason for the choice of $k = 5$ in the reference regimes of Fig. 6; the cluster centroids of the flow-dependent model are shown in Fig. 7 and can be compared with those.

The other simplified models reproduce some, but not all, of the regimes. The stochastic-forcing model has a good correspondence with $k = 3$, but for higher class numbers the correspondence is lost. At $k = 5$ it reproduces clusters 1, 3 and 4 of the reference model (see Fig. 6). The constant-forcing model has a generally poorer ability to reproduce the reference regimes, but still succeeds for some. The most stable cluster that is found for the different choices of k is the equivalent of regime 3 of Fig. 6.

The results of this and of the preceding section provide enough evidence that the low-frequency part of the reference model can be reproduced by a 10-dimensional system, and that the phenomena associated with the trailing modes, mostly transient eddy interaction, can be parametrized by the use of the flow-dependent empirical closure.

5. Discussion and conclusion

It has been shown that, at least in the case of an intermediate complexity general circulation model in perpetual winter conditions, the low-frequency part of the model variability can be reproduced by a low-dimensional system. In this case the model of the low-frequency variability has 10 variables and 10 equations, and all other variables can be parametrized in terms of these.

In the last thirty years many studies have employed statistical techniques for computing this dimensionality, normally based on EOF decomposition or on the fit of the PDFs of given atmospheric indices and their expected theoretical PDFs. At the same time, simplified low-dimensional, but fairly unrealistic, models have been developed to explain the low-frequency variability phenomena.

The model built in this work is simple, low-dimensional and realistic. It does not represent a statistical diagnostic demonstration of the low-dimensionality of the dynamics, but rather a dynamical reproduction of its causes, that is, the construction of its equations.

The low-order model constructed in this paper is based on two fundamental ideas; on one side the projection of the equations onto an orthogonal base, each EOF having an empirical (data adaptive) relevance; on the other, the possibility of constructing a closure as a function of the independent variables chosen, following DV2000.

The projection, or truncation, on the EOF basis, corresponds to a choice of the independent variables of the model that is determined by the phenomena that have to be modelled, their spatial scale and their frequency domain. As mentioned, the EOFs are norm-dependent vectors. Selten (1995) compared the performance of an EOF truncated model using different norms to forecast the trajectory of the non-truncated model. Physically meaningful norms, such as the total energy norm or the potential enstrophy norm, have the interesting property of conserving the integral constant (energy or potential enstrophy) in the truncation. Selten (1995) showed that this property does not improve the forecast skill at a given truncation, with respect to the canonic norm, but rather worsens it. For this reason the canonic norm of the stream function fields was used in this study. Nevertheless, it cannot be ruled out that the non-conservation of energy in the low-dimensional model may create additional spurious variability in the long climatic run. The realism of the variability patterns of the low-order model makes such a possibility not very believable, but it is still a point that needs to be clarified. Ad hoc experimentation should probably be used.

The forcing term added as a closure to the reduced system of equations is essential to reproduce correctly the climate of the reference model. The forcing term is meant to include the effect of all the phenomena excluded by the truncation. In the case of the reference model of this study, these phenomena are mainly the effect of interaction between transient high-frequency eddies and the large-scale flow.

A comparison has been carried out between different formulations of the closure term: a time-constant term, a time-varying stochastic term, and a time-varying term expressed as a function of the resolved part of the flow. All three terms are parametrizations of the transient eddies, with different degrees of complexity. The flow-dependent forcing term was found to give the best results. With this forcing, the 10-dimensional model exhibited a mean state, a low-frequency variance and NH weather regimes almost identical to those of the reference model. The flow dependency of the forcing term proved to be essential especially for the reproduction of the regimes. The necessity for an eddy forcing parametrization in a low-order model was also revealed by Metz (1986) and Achatz and Branstator (1999). In the latter case, the parametrization was a linear regression of tendency errors on the state of the model. In fact, Metz (1989) showed that a covariance, albeit small, could be found between eddy flux convergence terms and the large-scale flow. With such a linear parametrization the Achatz and Branstator (1999) model showed a reasonable climatology as far as mean and variance were concerned.

The model built in this paper, because of its simplicity and relatively high degree of realism, is a valuable tool for the study of the dynamics of the extratropical low-frequency variability. Part II will describe its application to a number of classical questions that remain to be answered in the field. In particular the relation between weather regimes and large-scale equilibria of the flow will be addressed by searching for the stationary states of the model and comparing them with cluster analysis. The issue of the stability of the stationary states will also be addressed, with the aim of understanding the mechanisms of regime decay and of the possible transition between regimes.

Although the present work is carried out in a pure 'reference model' approach, there is a clear possibility of extending the low-order model approach to simulation of the real atmosphere. The low-order model is composed of a dynamical part, that is basically the reduced version of a pre-existing model, and an empirical part. The empirical part can be fitted from observed data rather than from reference-model integration, and a model of the real atmosphere can be built. Of course this is not straightforward due to the data sampling problem.

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