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Modal de Géophysique

Informal view of the baroclinic instability

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3.1 Vortex stretching

We already introduced the potential vorticity which is a key concept in geophysical fluid dynamics. Potential vorticity (PV) is defined as

$$q = \frac{\omega + f}{h}\,, \tag{3.1}$$

with f the Coriolis parameter (background rotation), $\omega = \partial_x v - \partial_y u$ the relative vorticity and h the thickness of the layer. In the absence of forcing and dissipation, potential vorticity is conserved

$$\frac{Dq}{Dt} = 0\,. \tag{3.2}$$

This means that we can 'tag' each fluid parcel with its potential vorticity value (potential vorticity is an active tracer). To illustrate this concept, let's consider a fluid at rest on a slopping topography. The topography is uniform in the x direction and we only plot in Fig. 3.1 a (y, z) section. We displace a fluid column initially at y_0 (and $\omega_0 = 0$) and put it at y_1. This fluid column keeps its initial potential vorticity $q_0 = f/h_0$ and because $h_0 < h_1$ it will acquire positive relative vorticity $\omega > 0$. This phenomena is called vortex stretching (vortex squeezing in the opposite case).

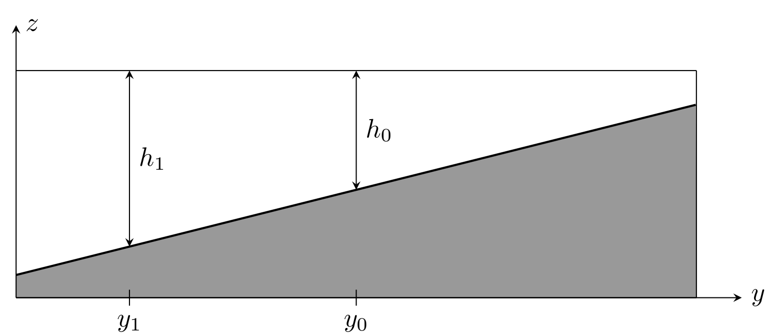

Figure 3.1: Conservation of Potential vorticity with a slopping bottom.

3.2 Rossby waves

We consider the same topography but we now look at what happens in the (x, y) plane (cf. Fig. 3.2). We move all the fluid parcels that are initially at rest at $y = y_0$ (dashed line) to the sine curve (blue curve). From the conservation of potential vorticity principle, we know that fluid parcels that are displaced in a shallower area will acquire negative relative vorticity (A and E) and fluid parcels that are displaced in a deeper area will acquire positive relative vorticity (C). Vortices A and C entrain the surrounding water parcels and their combined effect creates a flow field in B toward the deeper part. Similarly, the combined effect of C and E will produce a flow field in D towards the shallower part of the domain. After some time, the water parcels B and D will move to the deep and shallow area respectively, whereas the water parcels A, C, and E will move back to their initial position ($y = y_0$). It results that the blue curve will transform in the blue dashed curve. The main effect is thus a propagating wave in the direction of decreasing x. Note that fluid parcels only move along the y axis and do not propagate along the x axis. If there is a uniform flow with constant velocity U, the propagation speed c of the wave is Doppler shifted and if $U + c = 0$ (the wave propagation speed matches exactly the mean flow velocity), then we get a standing Rossby wave.

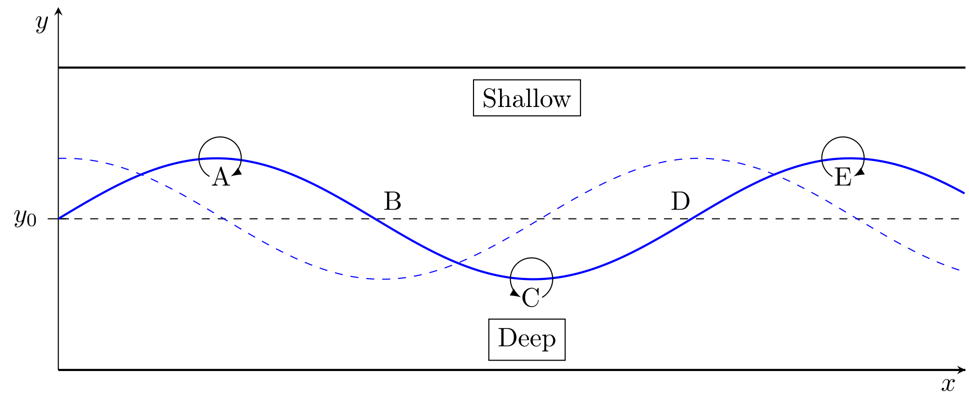

Figure 3.2: Rossby wave mechanism

3.3 Baroclinic instability in a two layer system

Suppose now that instead of the situation depicted in Fig. 3.1, we have a two layer system where the topography is replaced by a denser fluid. In each layer, the potential vorticity is conserved

$$q_u = \frac{\omega + f}{h_u} \quad \text{and} \quad q_l = \frac{\omega + f}{h_l}\,, \tag{3.3}$$

where q_u and q_l are the potential vorticity of the upper and lower layer respectively. Rossby waves in the upper layer will propagate according to the mechanism described in the previous section. In the lower layer, the mechanism is the same but the shallow and deep parts are interchanged which means we can draw a similar picture as in Fig. 3.2 for the lower layer but with the sign of the vortices reversed. The only difference with the one layer system is that the interface between the two layers is flexible (compared to the rigid topography). This means that if a fluid column in the upper layer is shifted from y_0 to y_1 (as in the example in Fig. 3.1), it will deform the interface so that the fluid column in the lower layer will stretch as well. In fact, both the upper and lower layer will acquire positive relative vorticity at the same location.

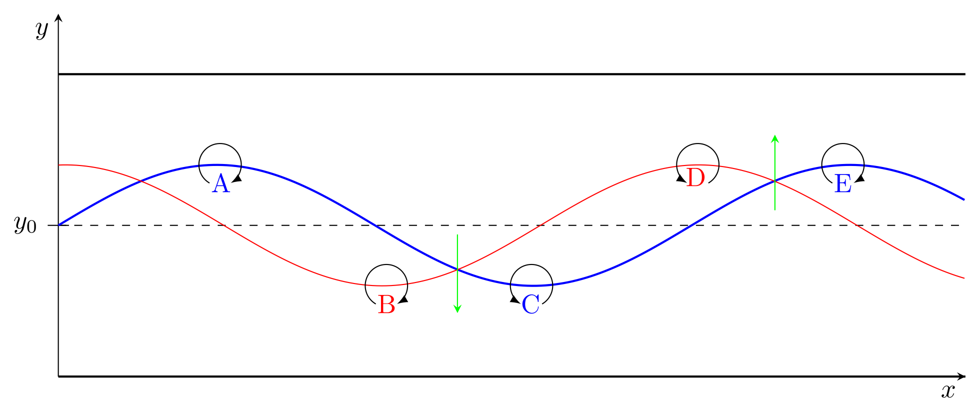

Figure 3.3: Baroclinic instability mechanism (blue: upper layer wave, red: lower layer wave).

Suppose that at some point, we reach the situation depicted in Fig. 3.3: a Rossby wave in the upper layer (blue) coexist with a Rossby wave in the lower layer (red) (B and D are now fluid columns of the lower layer). The coexistence of the vortices D and E will drive a flow towards $y > y_0$ between D and E (green arrow). Similarly, the vortices B and C will drive a flow towards $y < y_0$ (green arrow). This situation is unstable if the two waves remain in quadrature. This is possible because in a two layer system, a slanted interface in the y direction means that there is a vertical gradient of the flow according to the thermal wind relation

$$\frac{\partial u}{\partial z} = -\frac{g}{f\rho_0}\frac{\partial \rho}{\partial y}\,, \tag{3.4}$$

We write U_u and U_l the mean velocity in the upper and lower layer respectively. The mean flow in each layer will introduce a Doppler shift of the wave in each layer such that there always exists a situation where the wave in the upper and lower layer will be stationary with respect to each other. Infinitely small perturbations will grow and this situation is always unstable.

References

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