Ecole Polytechnique	**Printemps 2018**

Modal de Géophysique

Impact of rotation and stratification

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There are two factors that strongly impact the dynamics of the atmosphere and the oceans. First, the earth is spinning at a rate of one rotation per day. The rotating frame of reference is no longer Galilean and we must add forces to describe the dynamics of the flow. The second factor is that the earth receives incoming short wave radiations from the sun. These radiations heat the upper layer of the ocean and maintains the ocean in a stratified state. During this modal, we will study the effect of rotation and stratification on the dynamics of the flow. We will set up laboratory experiments and try to reproduce interesting phenomena that will give us intuition about what is going on in the ocean or in the atmosphere.

1.1 The influence of rotation

In a rotating frame, we must add two terms in the momentum equation to take into account the fact that we are not in a Galilean frame of reference: the Coriolis force and the centrifugal force.

$$\frac{D\boldsymbol{u}}{Dt} + \boldsymbol{f} \times \boldsymbol{u} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times r) = -\frac{1}{\rho}\boldsymbol{\nabla} P + \boldsymbol{g}\,, \tag{1.1}$$

with $\boldsymbol{f} = 2\Omega$ the Coriolis parameter. Can you give an order of magnitude for all these terms in the ocean and in the atmosphere?

L (m)	U (m s^{-1})	H (m)	W (m s^{-1})	ρ (kg m^{-3})	g (m s^{-2})	Ω (s^{-1})

1.1.1 The centrifugal force

1. To illustrate the role of the centrifugal force, you can observe what happens to a marble if we put it on a rotating table.
2. Now Fill up a tank of water and put it on the rotating table. Set the rotation rate to 10 rpm and wait until you think it is in solid body rotation. What are the differences between the rotating and non rotating states?
3. What is the balance of forces in Eq. (1.1)?
4. Can you get an equation for the free surface height as a function of the radius?

1.1.2 The Coriolis force

As you may have noticed form the dimensional analysis, the main balance of forces for a moving fluid is between the Coriolis force and the pressure gradient

$$\boldsymbol{f} \times \boldsymbol{u} = -\frac{1}{\rho}\boldsymbol{\nabla} P\,. \tag{1.2}$$

This equation is called the geostrophic balance and is the main equation that is used to diagnose a velocity field from a pressure field in the ocean or in the atmosphere. For a homogeneous fluid of uniform density, one can use this equation to prove that the flow dynamics will only occur in the plan perpendicular to the axis of rotation. (take the curl of Eq. (1.2)) and use the identity $\nabla \times (A \times B) = A(\nabla \cdot B) - (A \cdot \nabla)B + (B \cdot \nabla)A - B(\nabla \cdot A)$. This result is called the Taylor-Proudman theorem.

1. In a tank, place an small object ($\sim$ 2 cm). Fill up the tank so that the object is completely submerged. Turn on the rotation and wait until the fluid reaches solid body rotation. Inject dye at mid-depth and near the surface. Decrease the rotation rate to create a flow over the obstacle. How is the obstacle affecting the flow?

1.2 The stratification

Stratification is the second key element that affects the flow. The main effect of stratification is to inhibit vertical motion. In fact for a stably stratified flow, if we move a water parcel upward it will be heavier than the surrounding parcels and it will fall back down (This is an expression of the archimede's principle). It may overshoot its original position and arrive lower than its initial position. It will then rise up again. The frequency of this oscillation is named the Brunt-Vaisala frequency

$$N^2 = -\frac{g}{\rho_0}\frac{d\rho}{dz}\,. \tag{1.3}$$

1. In the room with the record players, fill up the first two tanks with tap water, and the last two tanks with a linear stratification (double bucket method). Start the rotation for the second and last tanks and let them reach solid body rotation. Inject dye in the first tank and observe how the dye mixes with the surroundings. Is there a preferred mixing direction?
2. Same question for the second, third and fourth tanks.
3. Can you explain the differences that you observe between the tanks.
4. From the combined effect of rotation and stratification emerges a length scale: the deformation radius

   $$R_d = \frac{NH}{f}\,. \tag{1.4}$$

   Redo the previous experiment with several values of f and N. Do you see a qualitative evolution of R_d (be careful when you inject the dye)
5. What is deformation radius in the ocean? in the atmosphere?

1.3 Geostrophic adjustment and Baroclinic instability

We now focus on an experimental set up with heating in the middle of the tank, that can be used to interpret some of the features of the atmospheric/oceanic general circulation. The purpose is to see how the dynamics evolves as a function of the rotation rate and to have a look at the non linear regime that develops. You can do a first experiment with a rotation rate of 5 rpm, one at 11 rpm and one at 18 rpm.

Fill up the tank with tap water and put an empty metal jar in the middle of the tank. Start the rotation and wait until you reach solid body rotation. Next prepare two syringes of dye, prepare 1 l of boiling water. Start the camera (with the remote control), fill up the jar with boiling water and put a few droplets of dye next to jar. Visualize the current and its shear by looking at the dye evolution.

1. Can you characterize the initial regime with the help of the thermal wind equations?
2. What is the length scale of the meanders? Is it related to any other length scale that you know?

1.4 Other experiments

There are a lot of experiments that you can do in a rotating tank: open `weathertank.mit.edu` and look at all the examples there.