Internal Waves, Mixing

I Internal waves

II Near-inertial oscillations

III Small-scale Mixing

While the first part reviewed a few processes of transport for which potential vorticity was the key variable, in this lecture we shall focus on processes of transport and mixing which possess ”fast time scales ” (when compared to $2\pi\ f^{-1}$,the local inertial period)

These fast time scales are often associated to small spatial scales although that is not always the rule [e.g. we shall consider below the case of Near-Inertial Oscillations that can have very large horizontal scales on the order of the atmospheric disturbances (O(1000km)].

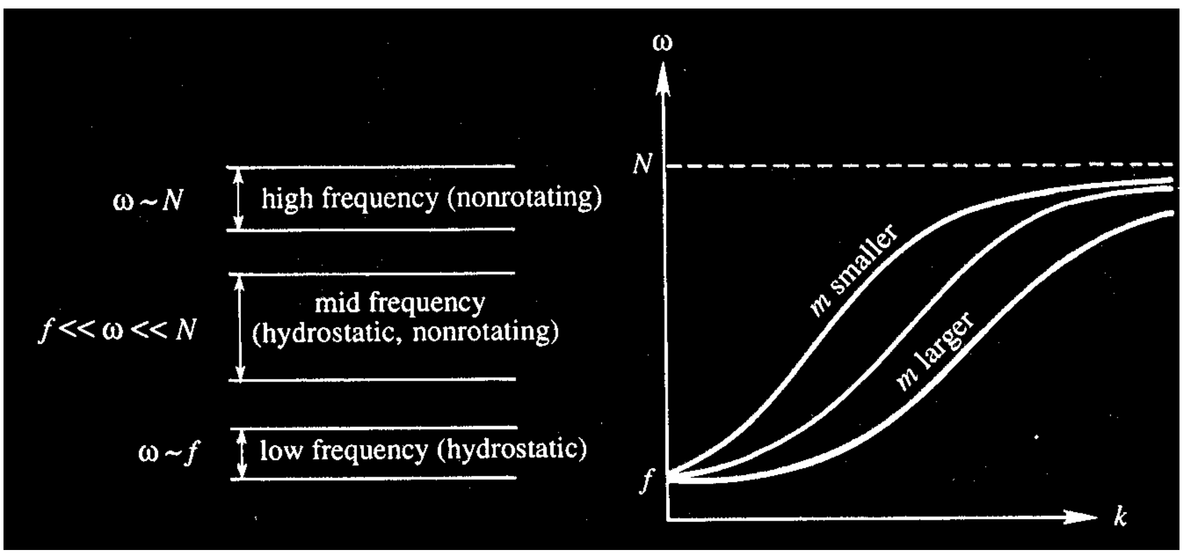
This will be a very selective account of some of the issues for which recent progress has been obtained.

Internal waves

Internal waves are ubiquituous in the ocean and in presence of the Coriolis force they can exist only in the frequency range ω such that

$$f < \omega < N$$

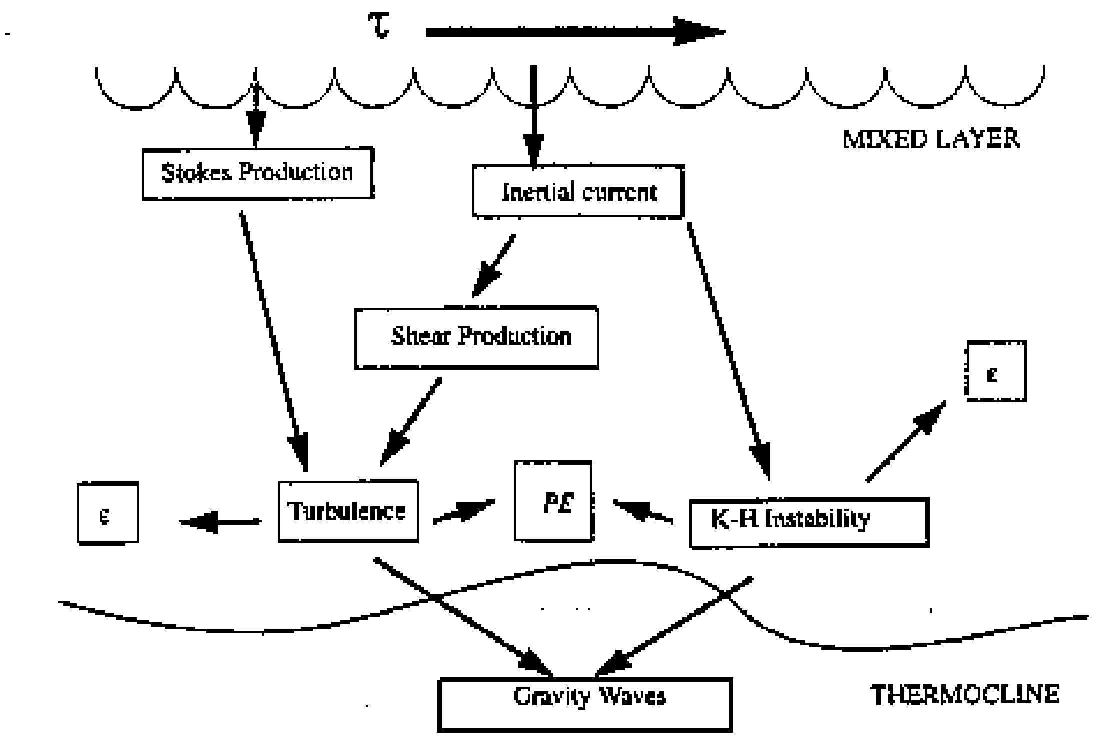
where N is Vaissala frequency and f the Coriolis parameter.



Different regimes and dispersion relation for internal waves

The first part of the talk will be concerned with the lowest frequency regime, while the small-scale abyssal mixing is most likely dominated by mid and high frequencies.

Near-inertial oscillations (NIOs)



Main pathways for energy transferred to the upper ocean via the surface wind stress

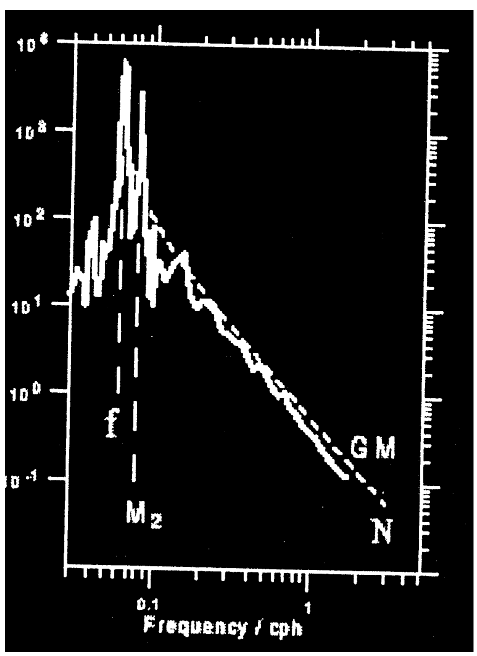
- NIOs excited by the large-scale wind-stress exerted on the ocean : ”ringing”
- inertial peak of the internal wave spectrum contains about half of the total kinetic energy of the oceans and an even larger fraction of $\overline{(\partial_z V)^2}$.
- Vertical propagating NIOs escape from the base of the Mixed layer into regions where N^2 is weaker, and thus the NIO shear might reduce the Richardson number below $1/4$ and trigger mixing events.

– A problem with the scenario described above is that NIOs with small horizontal wavenumbers propagate extrely slowly. Gill (1984) estimated that an NIO with horiz (typical of the atmospheric forcing mechanism) will radiate out of the mixed layer on time scales of one year or longer.

– However the OCEAN STORMS experiment clearly showed that after a storm NIO activity returns to background levels in about 20 days (D'Asaro et al. 1995)

– What mechanisms reduce the NIOs typical length scales such that they can propagate downward so quickly？

Answer ：The spatial modulation by the mesoscale eddy field. (Kunze, 1985)



Frequency spectrum of horizontal velocity at 140m depth during Ocean Storms Experiment

Order of magnitude and time scale of the effects of the mesoscale flow

Mesoscale flow : $Ro(\equiv \frac{U}{fL}) \;<<\; 1\;,\;\; Bu(\equiv \frac{N^2H^2}{f^2L^2}) \;=\; O$ (1)

$$\omega^2 \;\approx\; f^2(1 + \quad \frac{\zeta}{f} + \quad \frac{N^2k^2}{f^2m^2}\;)$$

Vorticity effects Dispersion effects

(Ro) (Ro)

$$\omega \;\approx\; f + \quad \frac{\zeta}{2} + \quad \frac{fk^2R_m^2}{2}$$

where R_m is the radius of deformation of vertical mode m

It assumes that :

– NIO horizontal length scale and velocity scale are the same as that of the mesoscale flow ;

– NIO vertical scale is much smaller than that of the mesoscale flow.

Consequences :

– Effects of the mesoscale flow are of the order of Ro ;

– The resulting waves are still mostly inertial ($\omega \approx f(1+Ro)$) ;

– Effects of the mesoscale flow occur on a slow time scale :
T = O [(Ro f)$^{-1}$]

Physics involved in the spatial modulation by the vorticity field

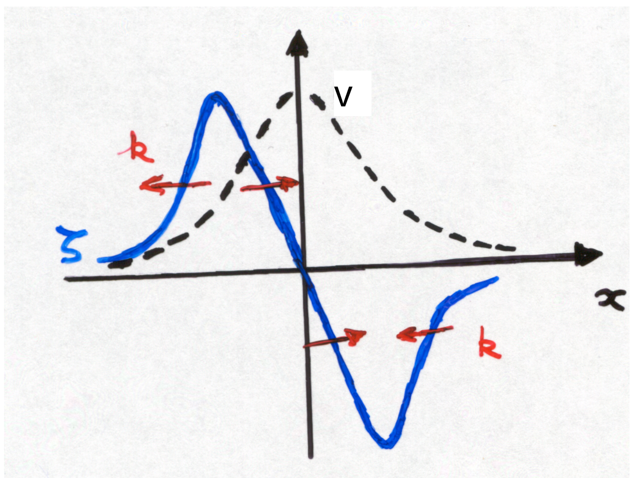
Solution with only the vorticity effects

– $u = u_o \sin((f + \frac{1}{2}\zeta)t)$

– using : $\zeta \approx \zeta_o + \frac{\partial \zeta}{\partial x}.x$

$u = u_o \sin((f + \frac{1}{2}\zeta_o)t - k.x) \;\; with \;\; k = -\frac{1}{2}\frac{\partial \zeta}{\partial x}.t$

$\Longrightarrow$ Spatial heterogeneity grows with time



Consequences , subsequent slow dispersion effects lead to :

– NIO concentration in the $\zeta < 0$ regions ;

– NIO depletion in the $\zeta > 0$ regions.

NIO equations (Young and Ben Jelloul, 1997, YBJ97)

YBJ97 have reformulated the problem of the advective distorsion by mesoscale eddies leading to the decrease of their coherence scale, using a multiple time scales method.

The NIOs are expressed as

$$\begin{aligned} u + iv &= e^{-if_0 t} LA, \\ w &= -\tfrac{1}{2} f_0^2 N^{-2} (A_{zx} - iA_{yz}) e^{-if_0 t} + c.c., \\ b &= \tfrac{i}{2} f_0 (A_{zx} - iA_{yz}) e^{-if_0 t} + c.c., \\ p &= \tfrac{i}{2} f_0 (A_x - iA_y) e^{-if_0 t} + c.c., \end{aligned}$$

with $i^2 = -1$ and where L is a differential operator defined by

$$LA = (f_0^2 N^{-2} A_z)_z.$$

$N(z)$ is the buoyancy frequency and f_0 the inertial frequency.
The NIO equations in dimensional form :

$$\frac{\partial}{\partial t} LA \quad + J(\psi, LA) \quad + \frac{if_0}{2} \nabla^2 A \quad + \frac{i}{2} \zeta LA \quad = 0,$$

advection by eddies   dispersion term   vorticity term

ψ and $\zeta \equiv \nabla^2 \psi$: streamfunction and relative vorticity of the mesoscale eddies (assumed to be geostrophic). J is the Jacobian operator.

YBJ97's equation for the NIOs capture the essential physics of the three-fold influence exerted by the geostrophic eddies :

(1) the advection

(2) the dispersion (corresponding to the $\zeta/2$ frequency shift of Kunze (1985))

(3) refraction effects (leading to the concentration of NIOs in anticyclonic vorticity regions).

Using higher order asymptotics, Reznick et al. (2001) have furthermore confirmed that there is no transfer of energy between NIOs and the geostrophic flow.

Spatial heterogeneity of NIOs (Klein and Llewellyn-Smith, 2001, KLS01)

Context :

Preceding studies have considered **isolated or monochromatic** mesoscale structures characterized by ONE ty pical length scale (or mostly one typical wavenumber). The build-up of the spatial heterogeneity of the NIOs is related to the INCREASE with time of their wavenumbers :

$$\frac{dk}{dt} \ \alpha \ -\nabla\zeta$$

KLS01 addressed the problem of the build-up of the spatial heterogeneity of the NIO when a turbulent mesoscale flow (characterized by a continuous wavenumber spectrum).

The QG flow field

The simulated background turbulent quasigeostrophic flow is forced by the baroclinic instability of a vertically sheared zonal flow $\overline{U}(z)$ and damped by a bottom Ekman layer (at $z = -H$).

The governing equation for the background flow is

$$\frac{\partial q}{\partial t} + \overline{U}\frac{\partial q}{\partial x} + J(\phi, q) + \beta\frac{\partial \phi}{\partial x} = M + D,$$

– ϕ is the perturbated QG streamfunction related to ψ :
  $\psi = -\overline{U}y + \phi$.

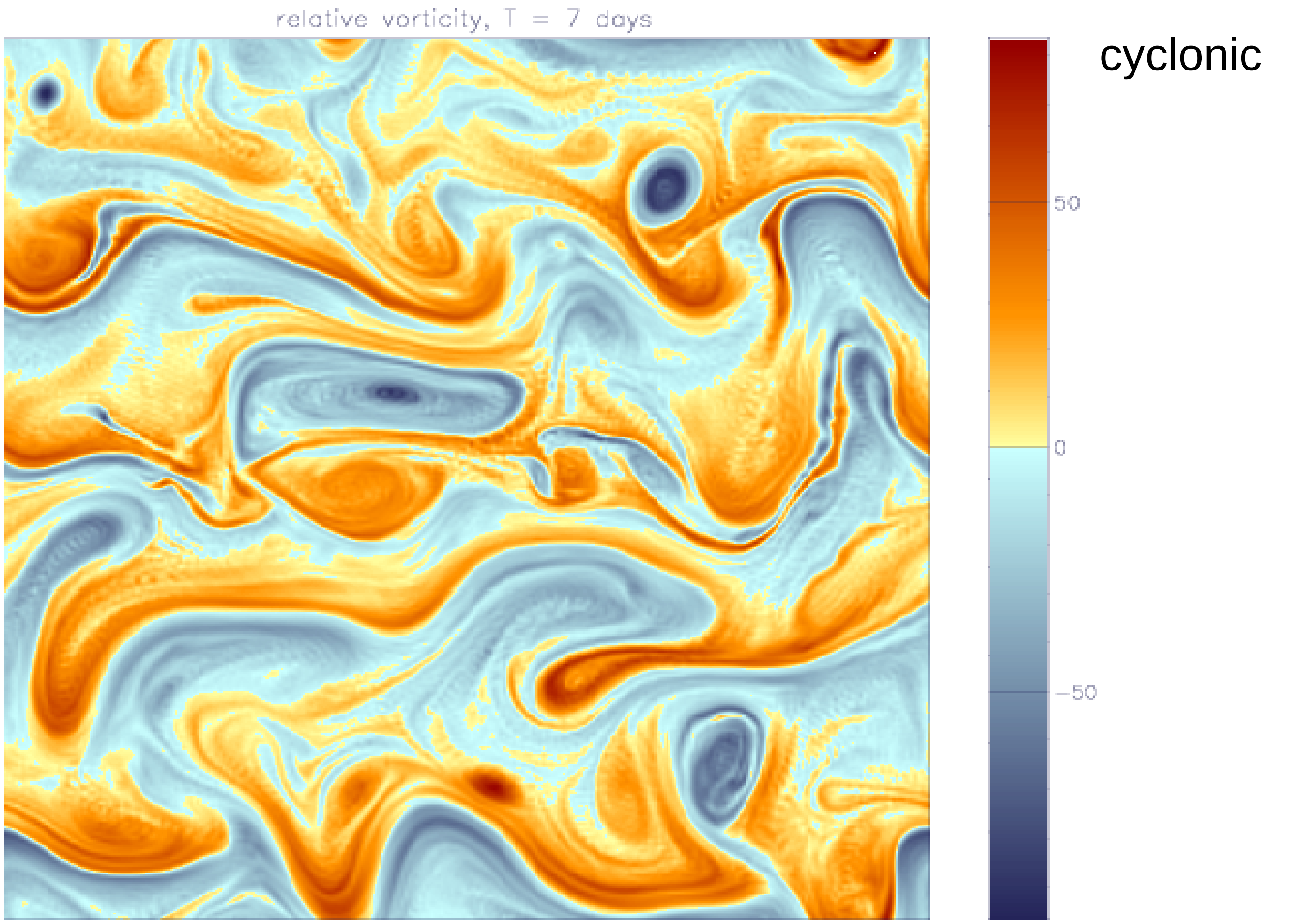
– $q \equiv \nabla^2\phi + (f_0^2 N^{-2}\phi_z)_z$ is the perturbated potential vorticity.

– M is the forcing term and D encompasses dissipative terms.

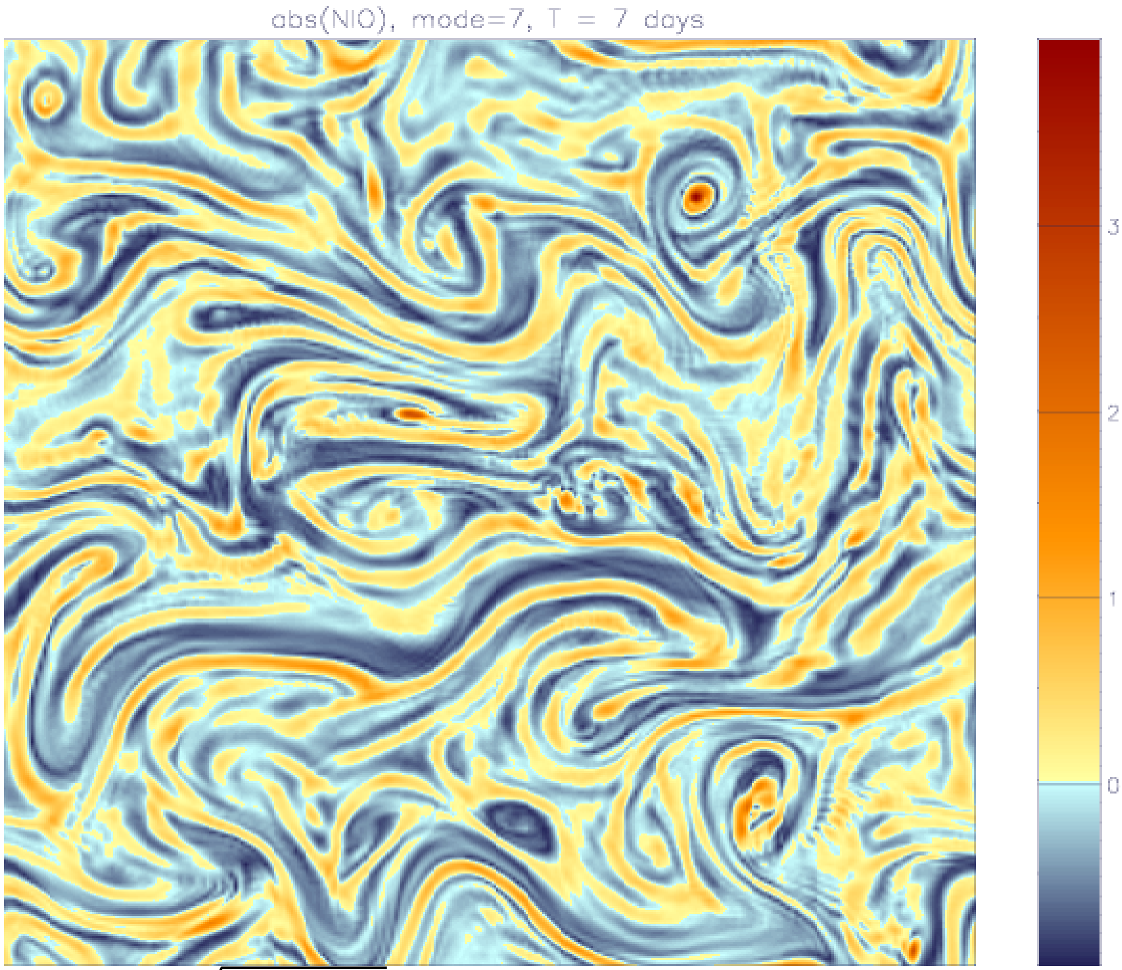
Typical time scale of the mesoscale flow is $T = 30 days$, first radius of deformation is 50 km, spatial domain size $\approx$ 2200 km.

Resolution 256 x 256 x 8 vertical modes.

They integrated the NIOs equation of YBJ97 advected by the ϕ output obtained by quasigeostrophic simulation. Initial conditions for the NIO is a largest scale sinusoidal field and all vertical modes have the same amplitude.



vorticity at a depth of 100 m after 7 days.



NIO speed $\sqrt{u^2 + v^2}$ associated with the 7th vertical mode at the same depth and time

SUMMARY OF KLS01 :

In a QG turbulent flow field, NIO dispersion is such that :

– The trapping (refraction) regime dominates and its spatial heterogeneity is close to the vorticity field at a given time.

– The geometry of the NIO energetic structures should match the mesoscale ζ-structures (for the low vertical modes) and the strong ζ-fronts (for the higher vertical modes).

Consequences for Mixing

- Concentration of NIO activity will trigger mixing events

- Strong spatial heterogeneity of Mixing both horizontally and vertically

Small-scale Mixing

Context :

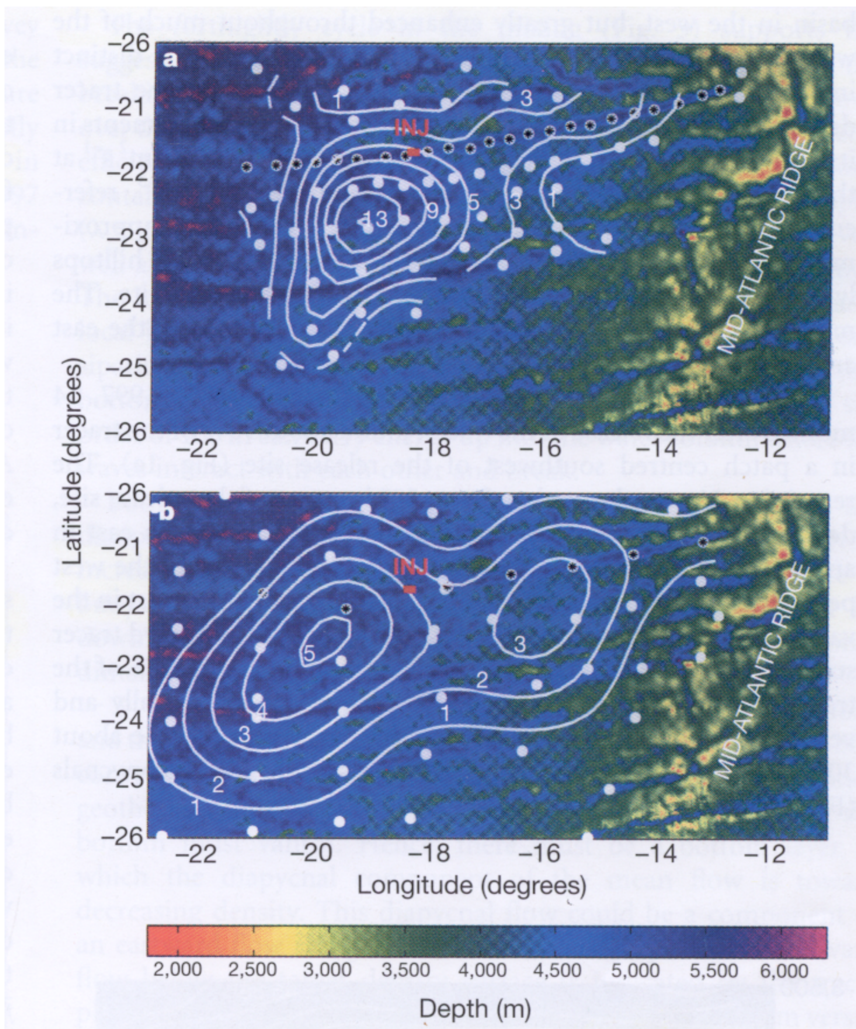
- The overturning thermohaline circulation of the ocean plays an important role in modulating the Earth's climate. But whereas the mechanisms for the vertical transport of water into the deep ocean -deep water formation at high latitudes-, has been largely identified, it is not clear how the compensating vertical transport of water from the depths to the surface is accomplished.
- Turbulent mixing across isopycnic surfaces can reduce the density of water and enable it to rise and is therefore important to quantify.
- However, measurements of the internal wave field, the main source of energy for mixing, and of turbulent dissipation rates, have typically implied diffusivities across isopycnic surfaces of only $\approx 10^{-5} m^2 s^{-1}$, too small to account for the return flow.
- Historically, inferences of the value of the vertical diffusivity from Munk's (1966) "abyssal recipes" or by performing budgets of heat for abyssal basins (Whitehead, 1989) have led to values of κ_v of $10^{-4} m^2 s^{-1}$. Such inferences are based on postulated simple advection-diffusion balances for heat, salinity, or a given passive tracer field e.g. ^{14}C.

Purposeful Tracer release experiments (Ledwell et al. 2000)

Deep Brazil Basin experiment

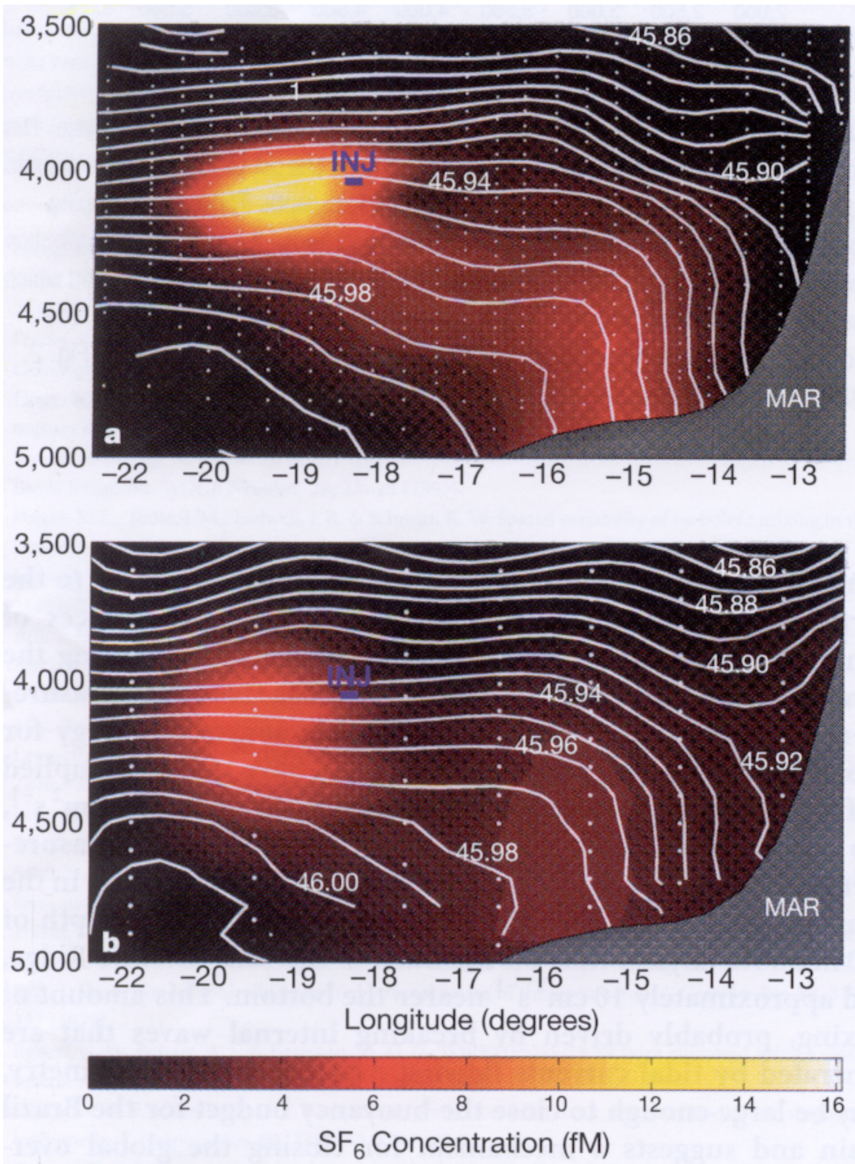
Both turbulent microstruce survey and release of SF6 (sulphur hexafluoride) above one zonal valley on the flank of the Mid-Atlantic Ridge at an average depth of 4000m, [1000m above the valley floor and 500m above the hilltops].

Microstructure data showed small diapycnal mixing in the West over the smooth valley and greatly enhanced diapycnal mixing in the East, with a further increase toward the bottom.



Tracer distribution (a) 14 months after release (b) 26 months after release. Colours denote bottom depth

Tracer to the west peaked near the target density surface, while tracer to the east was concentrated nearer the bottom. In the last survey, 30% of the tracer found is located in the eastern patch that has dispersed strongly across isopcnals.

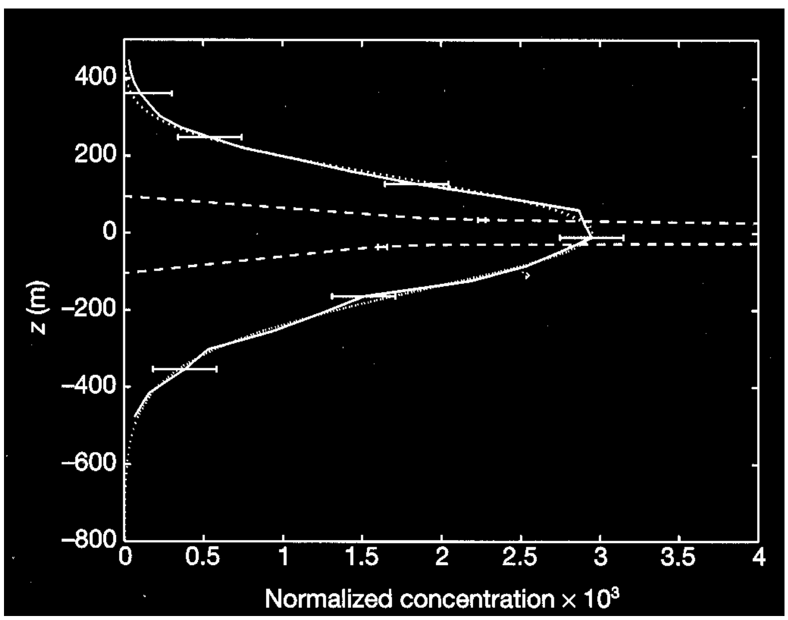


Vertical sections of tracer concentration and potential density for the same times. The label 'INJ' marks the release site of the tracer

Both horizontal and vertical sections suggest that diapycnal mixing increases toward the east as density surfaces approach the bottom.

The inventory of tracer as a function of potential density has been transformed into a profile of concentration versus height z above the target density surface of injection.

Diapycnal diffusivity $\kappa_{\text{v}}(z)$ was estimated by applying a one-dimensional model for the tracer evolution, yielding $\kappa = 3\ 10^{-4} m^2 s^{-1}$ at 4000 m depth and increasing to $8\ 10^{-4} m^2 s^{-1}$ at 4500 m depth, assuming a constant sinking rate of the tracer.

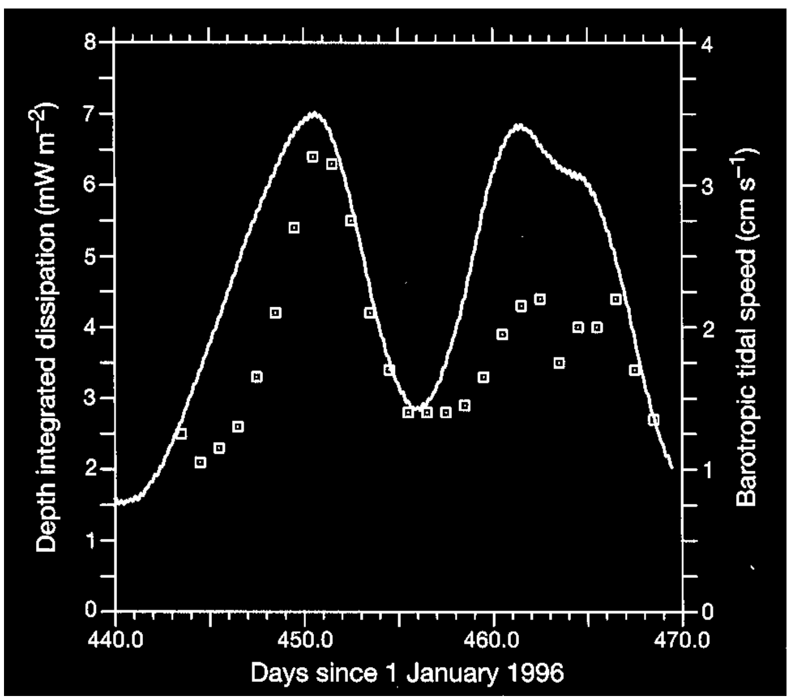


Mean vertical tracer profiles and model fit. Dashed lines depict the initial condition.

The above values agree in magnitude with the diffusivity inferred from the measurements of the dissipation of turbulent kinetic energy . Microstructure profiles clearly show an increase of diffusivity with decreasing height above the bottom.

Depth-averaged dissipation data document a fortnightly modulation in the mixing that lags the intensity of the barotropic tide by a few days.

This fortnightly cycle in the mixing supports the suggestion that the enhanced mixing in the area results from the breaking of internal waves generated as the barotropic tide flows over the rough ridge bathymetry.



Comparison of the turbulent dissipation rate and tidal speeds

Tidal flows over rough bathymetry generate upward-propagating internal waves with high shear at levels well above the bottom and this shear can feed turbulent mixing. Observations imply a spatially local balance between internal wave generation by tides and dissipation.

Summary of mixing by internal waves

Abyss:

- Enhanced diapycnal mixing over rough topography triggered by breaking of internal waves excited by barotropic tide.
- Order of magnitude of abyssal mixing coefficient compatible with water mass transformation.
- For further details on tidal mixing and dynamics of entrainment near boundaries:

  S.Legg's poster and Web page.

Thermocline:

- Little contribution from internal waves. Stirring and advection along isopycnal surfaces dominate

Surface layers:

- Near-Inertial Oscillations trapping and spatial heterogeneity triggers localized mixing events

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