École Doctorale des Sciences de l'Environnement d'Île-de-France

Année Universitaire 2014-2015

Modélisation Numérique de l'Écoulement Atmosphérique et Assimilation de Données

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Cours 3

20 Avril 2015

Fig. 1: Members of day 7 forecast of 500 hPa geopotential height for the ensemble originated from 25 January 1993.

Figure 6 Hurricane Katrina mean-sea-level-pressure (MSLP) analysis for 12 UTC of 29 August 2005 and t+84h high-resolution and EPS forecasts started at 00 UTC of 26 August:

1st row: *1[st] panel: MSLP analysis for 12 UTC of 29 Aug*
2[nd] panel: MSLP t+84h T_L511L60 forecast started at 00 UTC of 26 Aug
3[rd] panel: MSLP t+84h EPS-control T_L255L40 forecast started at 00 UTC of 26 Aug
Other rows: 50 EPS-perturbed T_L255L40 forecast started at 00 UTC of 26 Aug.

The contour interval is 5 hPa, with shading patters for MSLP values lower than 990 hPa.

Pourquoi les météorologistes ont-ils tant de peine à prédire le temps avec quelque certitude ? Pourquoi les chutes de pluie, les tempêtes elles-mêmes nous semblent-elles arriver au hasard, de sorte que bien des gens trouvent tout naturel de prier pour avoir la pluie ou le beau temps, alors qu'ils jugeraient ridicule de demander une éclipse par une prière ?[...] un dixième de degré en plus ou en moins en un point quelconque, le cyclone éclate ici et non pas là, et il étend ses ravages sur des contrées qu'il aurait épargnées. Si on avait connu ce dixième de degré, on aurait pu le savoir d'avance, mais les observations n'étaient ni assez serrées, ni assez précises, et c'est pour cela que tout semble dû à l'intervention du hasard.

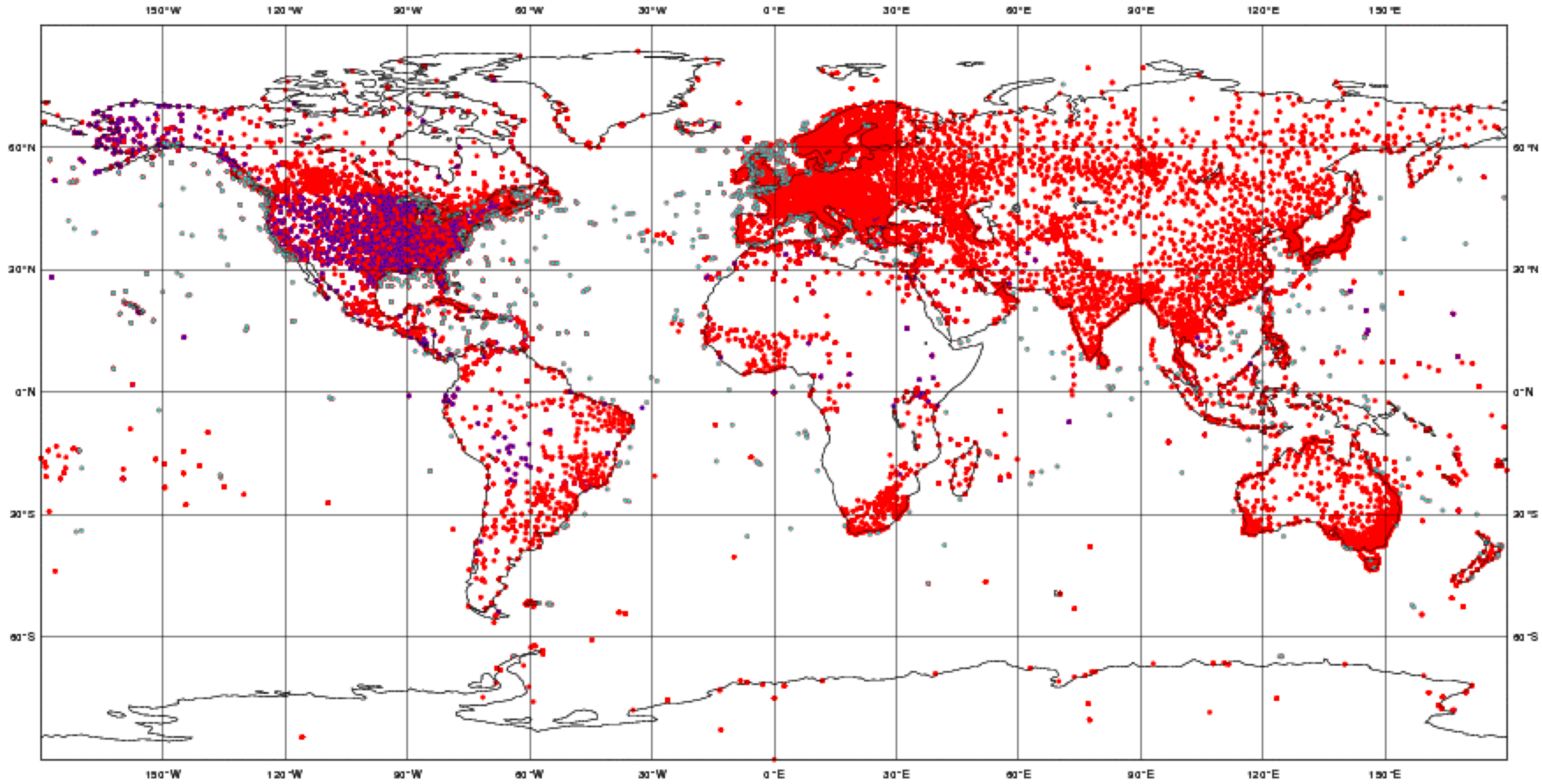
H. Poincaré, *Science et Méthode*, Paris, 1908

ECMWF Data Coverage (All obs DA) - Synop-Ship-Metar

19/Apr/2015; 00 UTC

Total number of obs = 63792

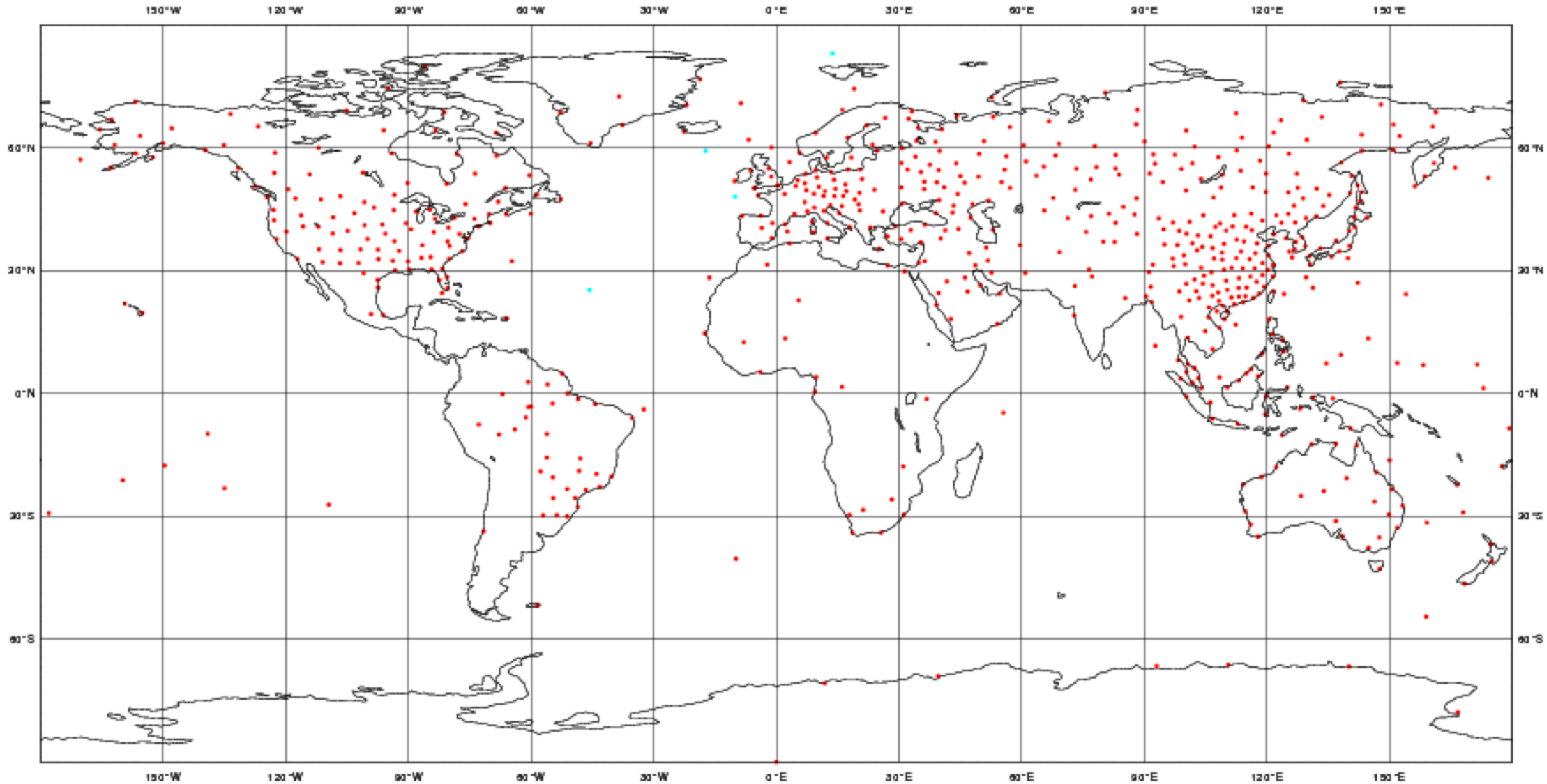
23929 SYNOP   3923 SHIP   35940 METAR



Magics 2.14.4 (64 bit)   ECMWF

ECMWF Data Coverage (All obs DA) - Temp
19/Apr/2015; 00 UTC
Total number of obs = 642

4	SHIP	0	MOBILE
638	LAND	0	DROPSONDE



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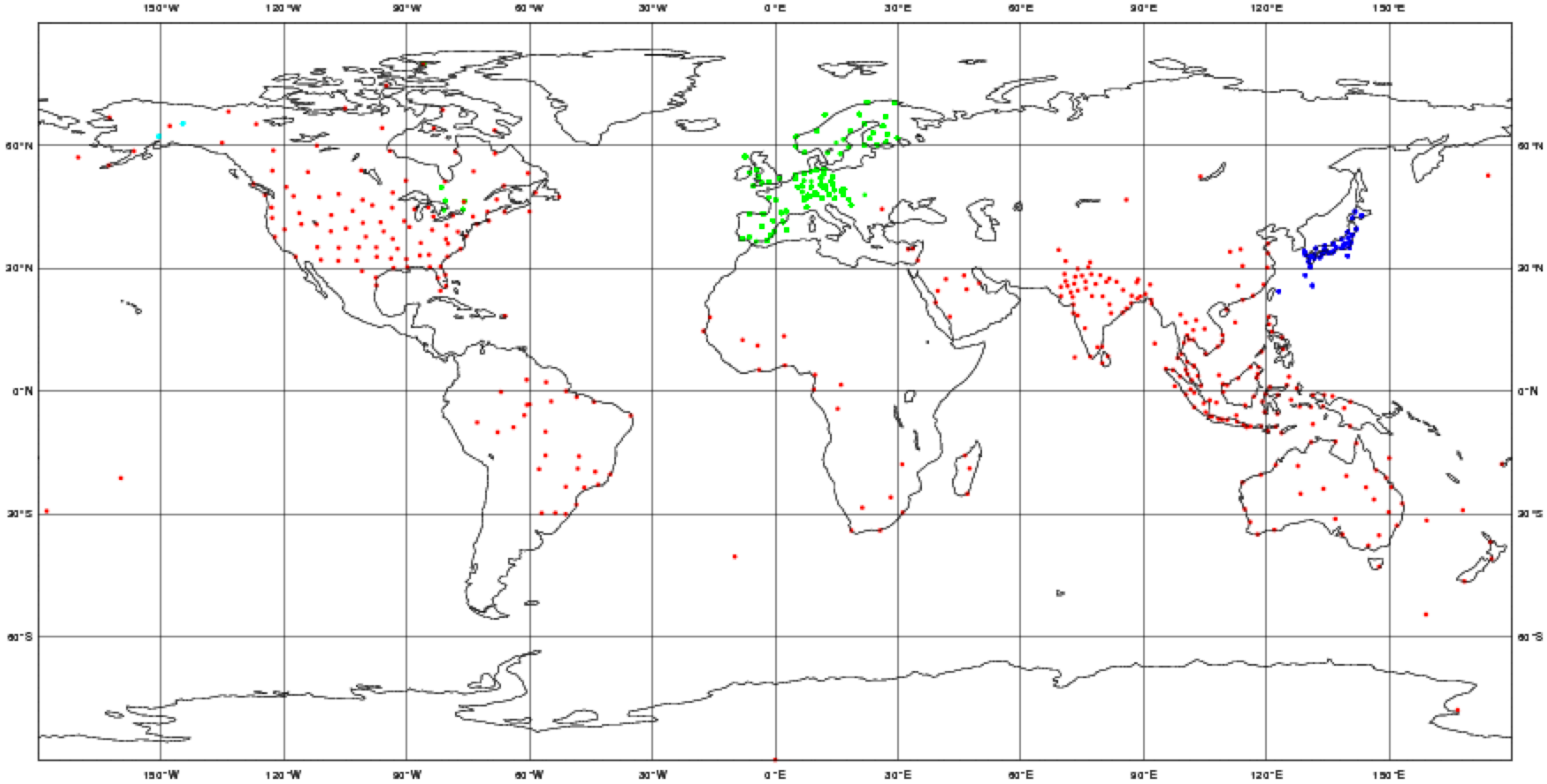
ECMWF

ECMWF Data Coverage (All obs DA) - Pilot-Profiler
19/Apr/2015; 00 UTC
Total number of obs = 2677

•	18 US-PROF	•	198 JP-PROF
•	357 PILOT	•	2104 EU-PROF



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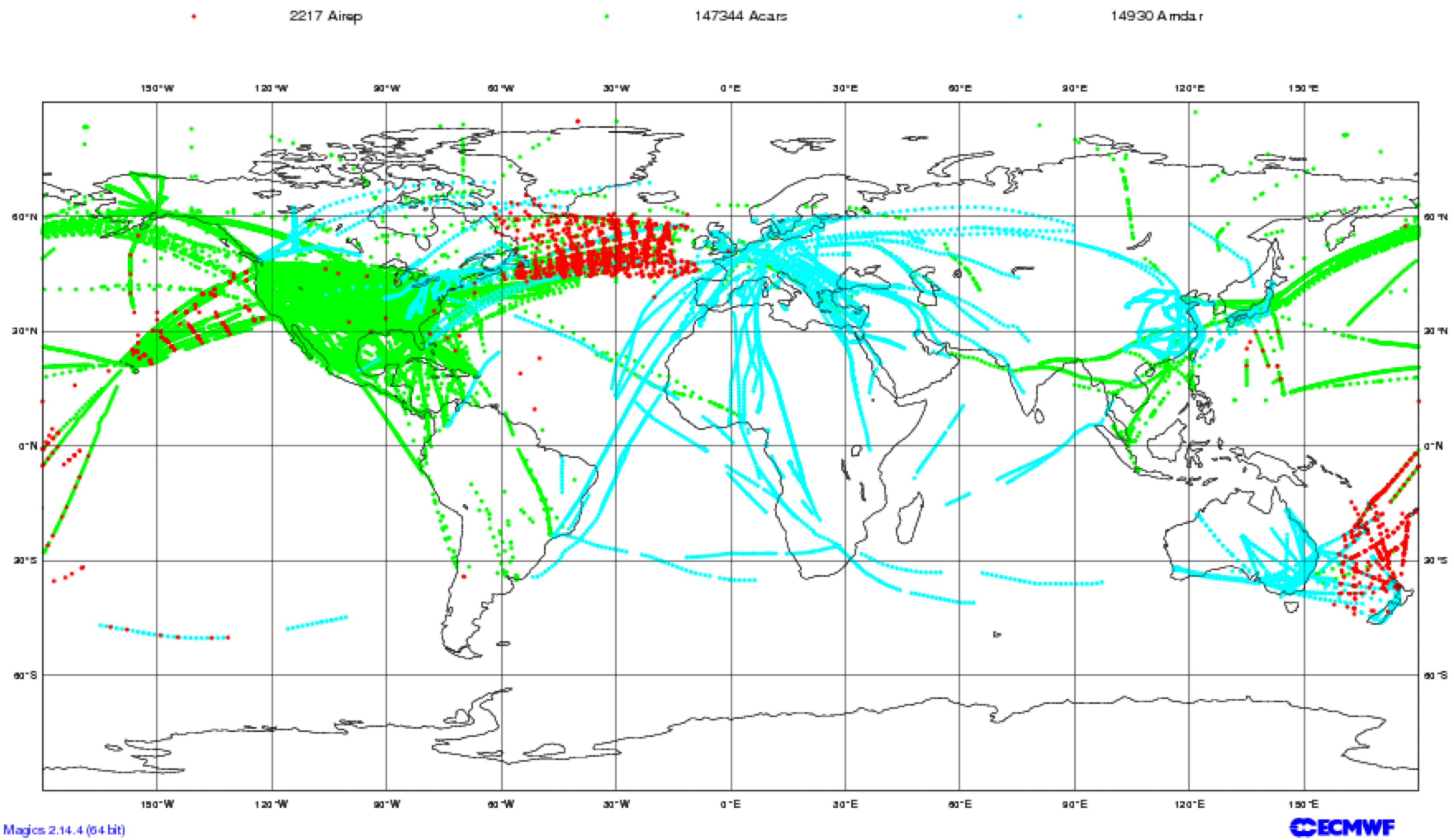
ECMWF

ECMWF Data Coverage (All obs DA) - Aircraft
19/Apr/2015; 00 UTC
Total number of obs = 164491

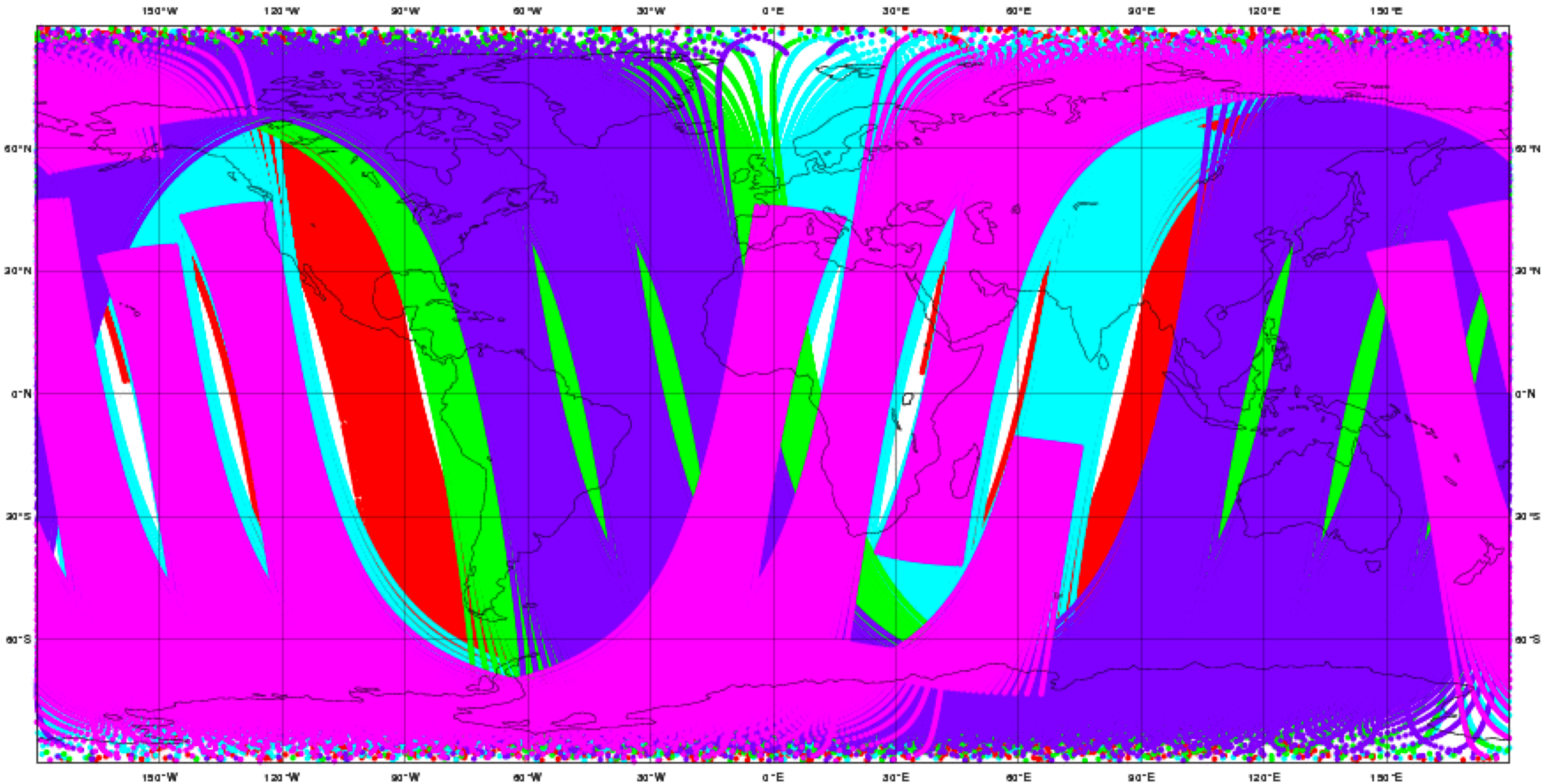


ECMWF Data Coverage (All obs DA) - AMSU-A

19/Apr/2015; 00 UTC

Total number of obs = 599550

• 83742 Noaa15 • 121411 Noaa18 • 135512 Noaa19 • 68692 AQUA • 109112 METOP-A • 81081 METOP-B



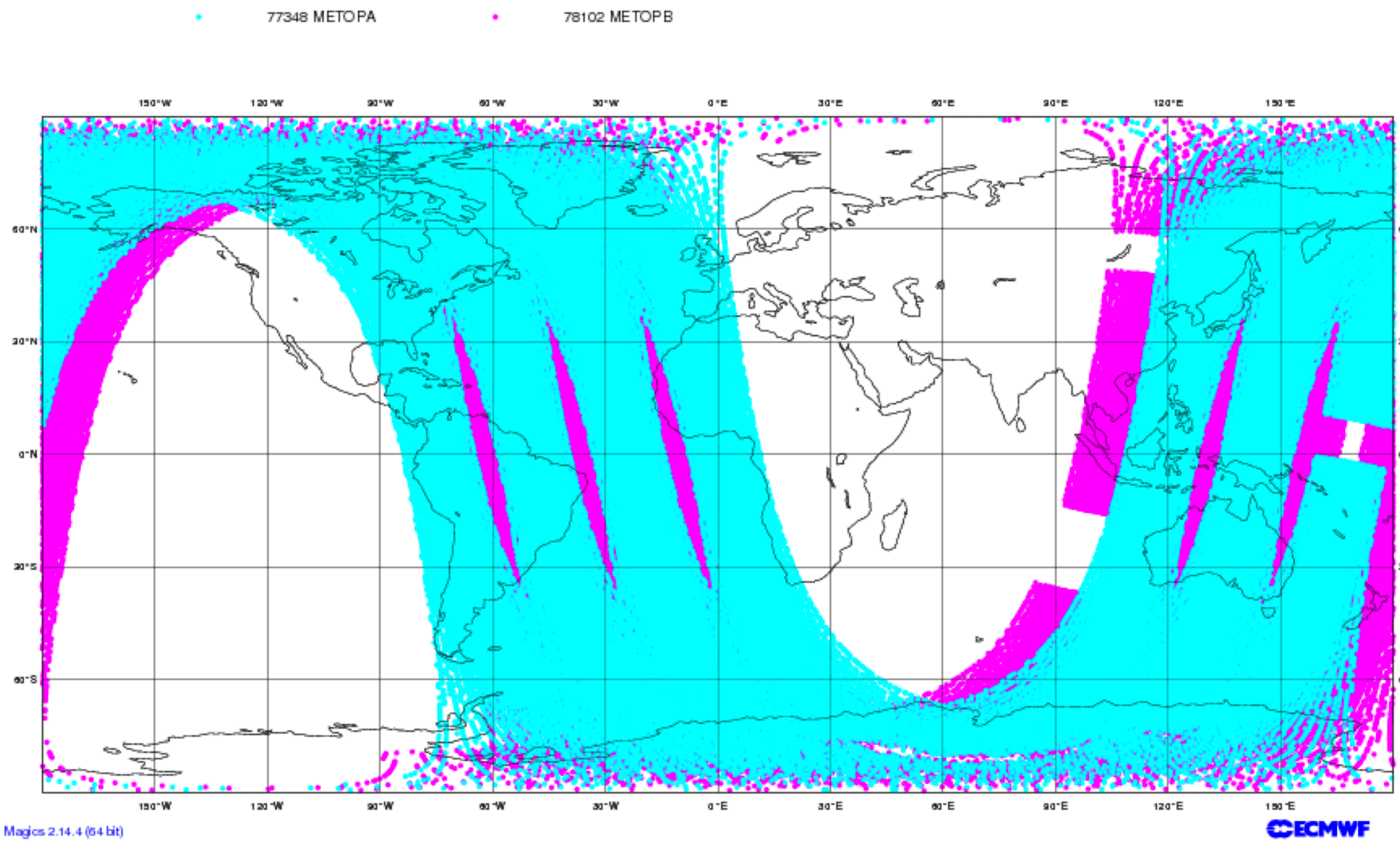
Magics 2.14.4 (64 bit)

ECMWF

ECMWF Data Coverage (All obs DA) - IASI

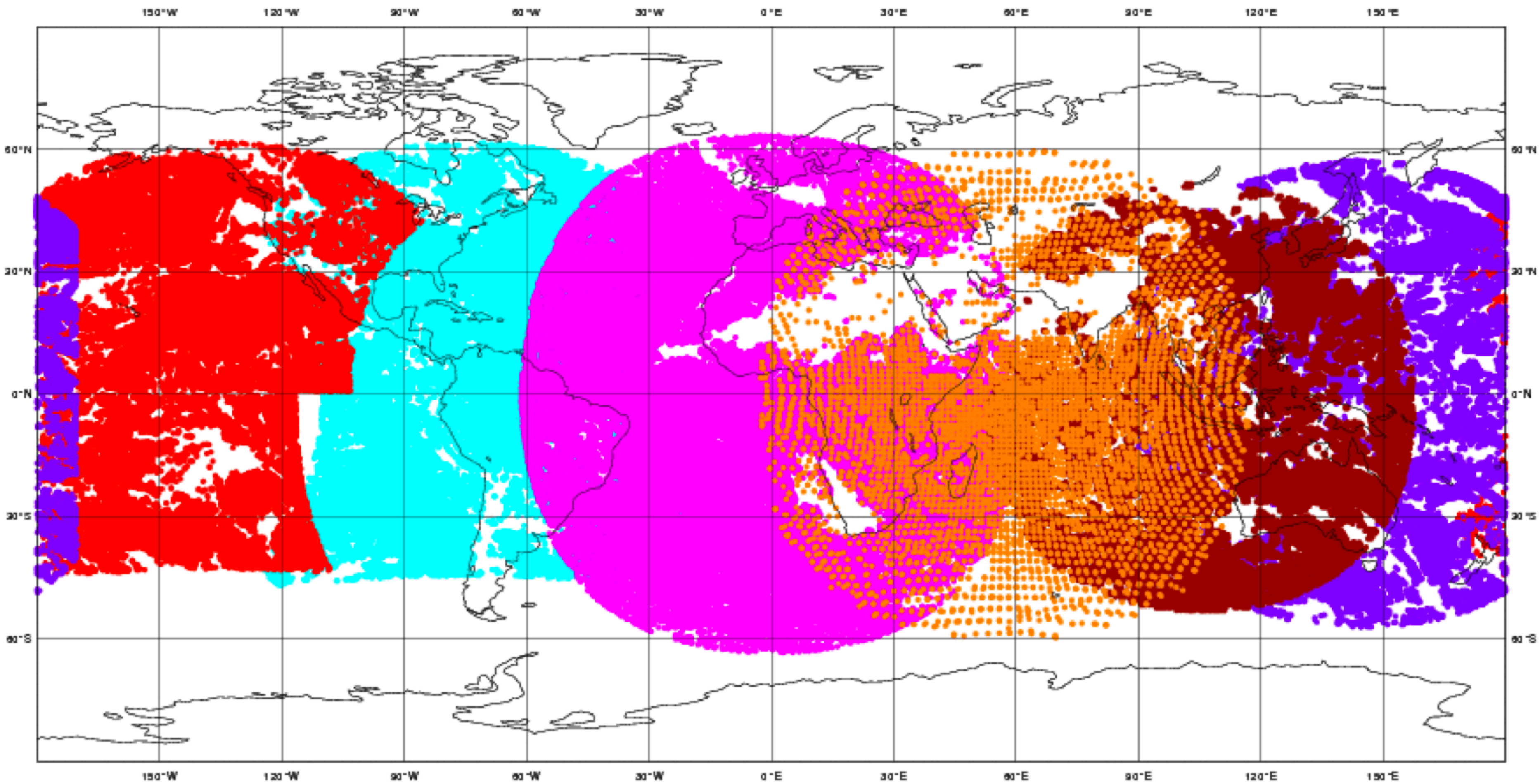
19/Apr/2015; 00 UTC

Total number of obs = 155450



ECMWF Data Coverage (All obs DA) - AMV IR
19/Apr/2015; 00 UTC
Total number of obs = 315968

60937 Goes15 · 121423 Goes13 · 64280 Met10 · 0 Mtsat-1R · 38689 Mtsat2 · 0 FY-2D · 23556 FY-2E · 7083 Met7 · 0 Goes14



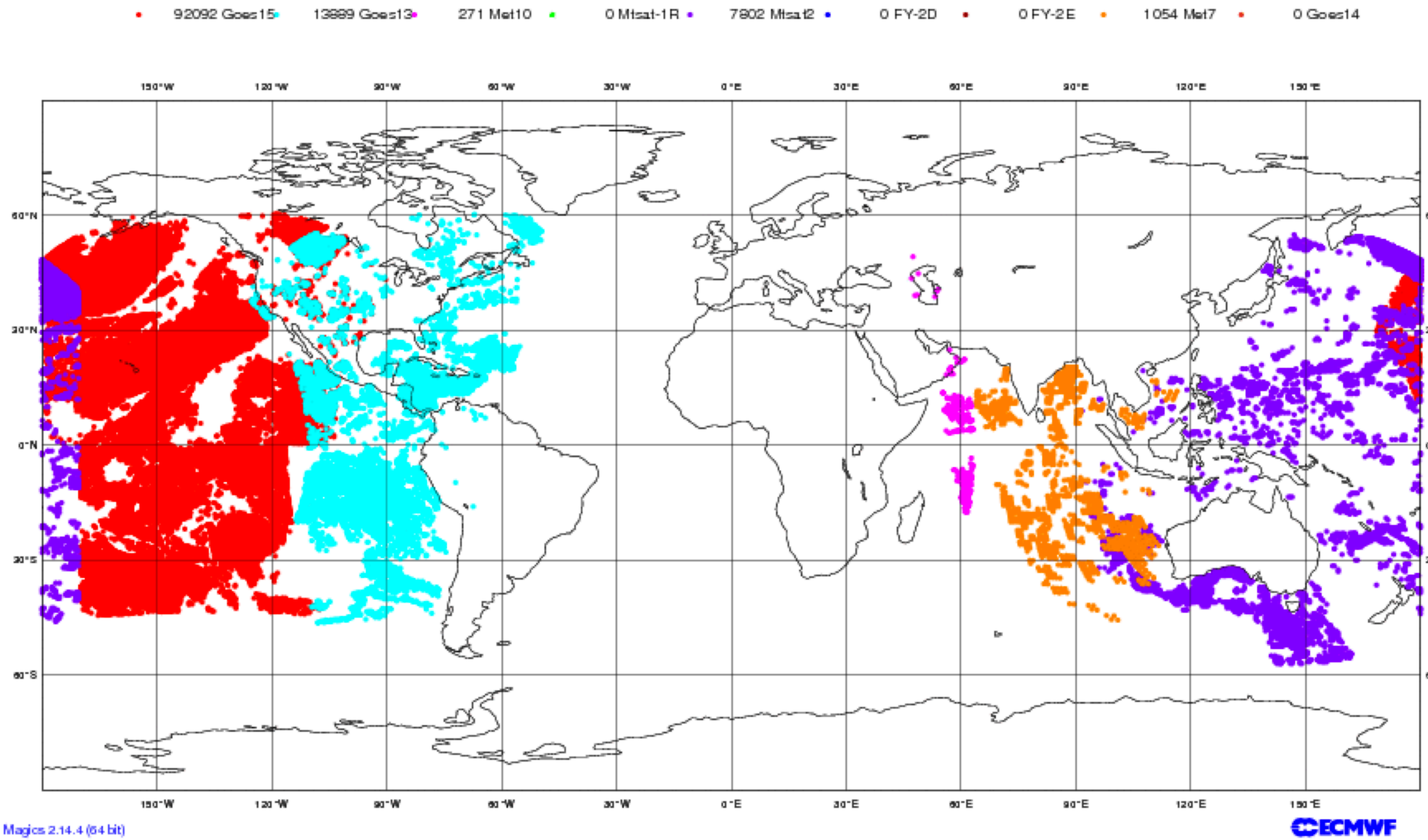
Magics 2.14.4 (64 bit)

ECMWF

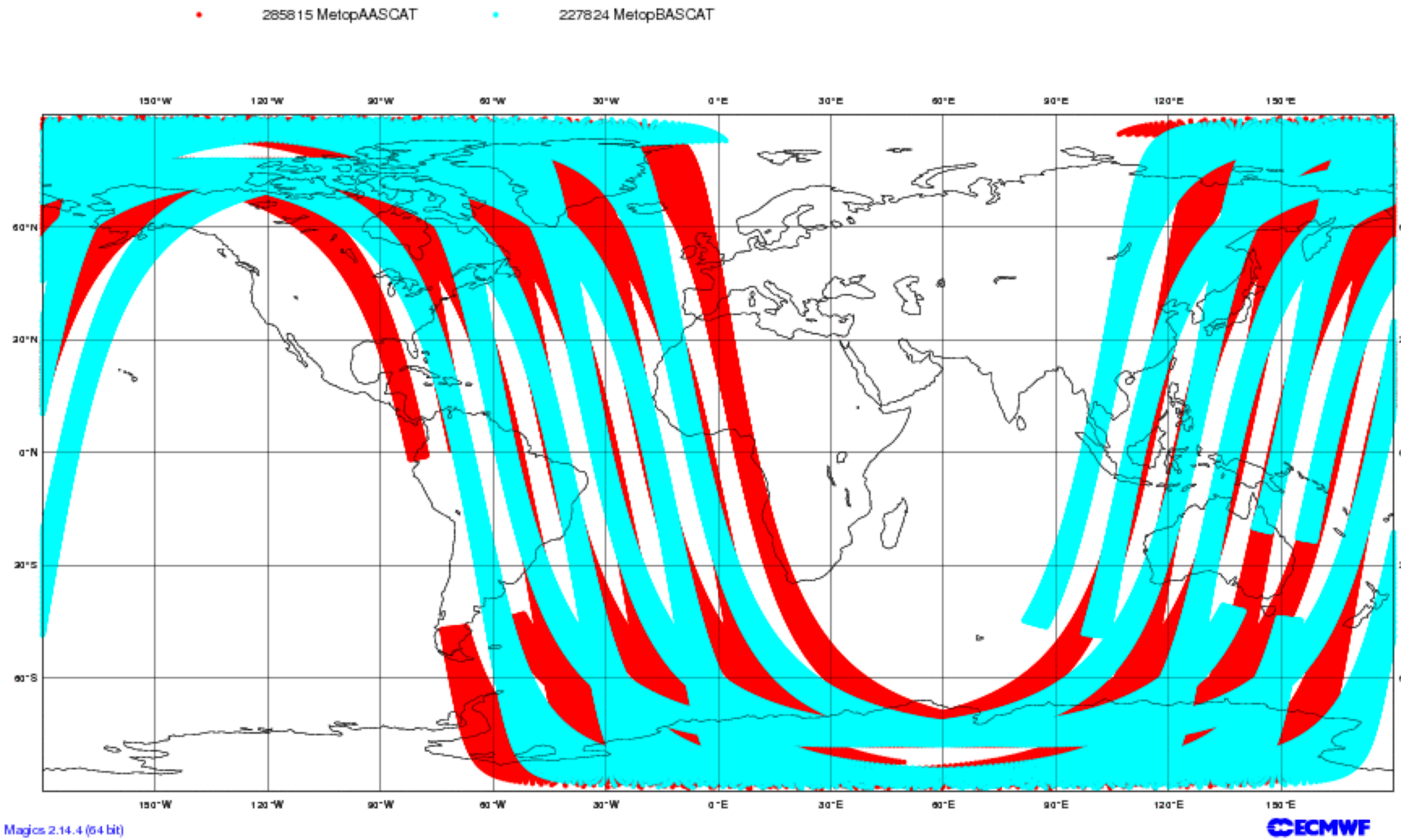
ECMWF Data Coverage (All obs DA) - AMV VIS

19/Apr/2015; 00 UTC

Total number of obs = 115108



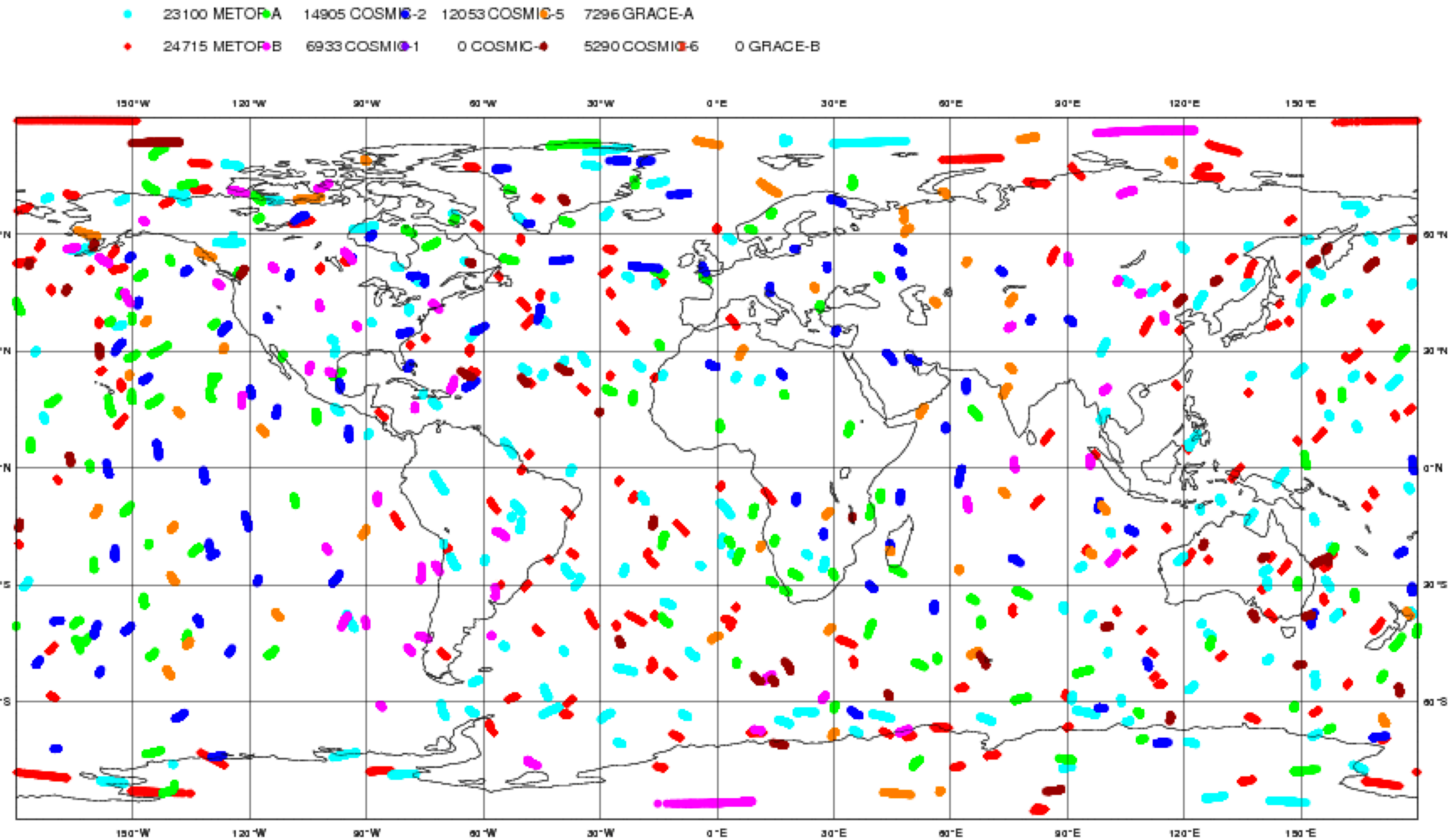
ECMWF Data Coverage (All obs DA) - SCAT
19/Apr/2015; 00 UTC
Total number of obs = 513639



ECMWF Data Coverage (All obs DA) - GPSRO

19/Apr/2015; 00 UTC

Total number of obs = 94292



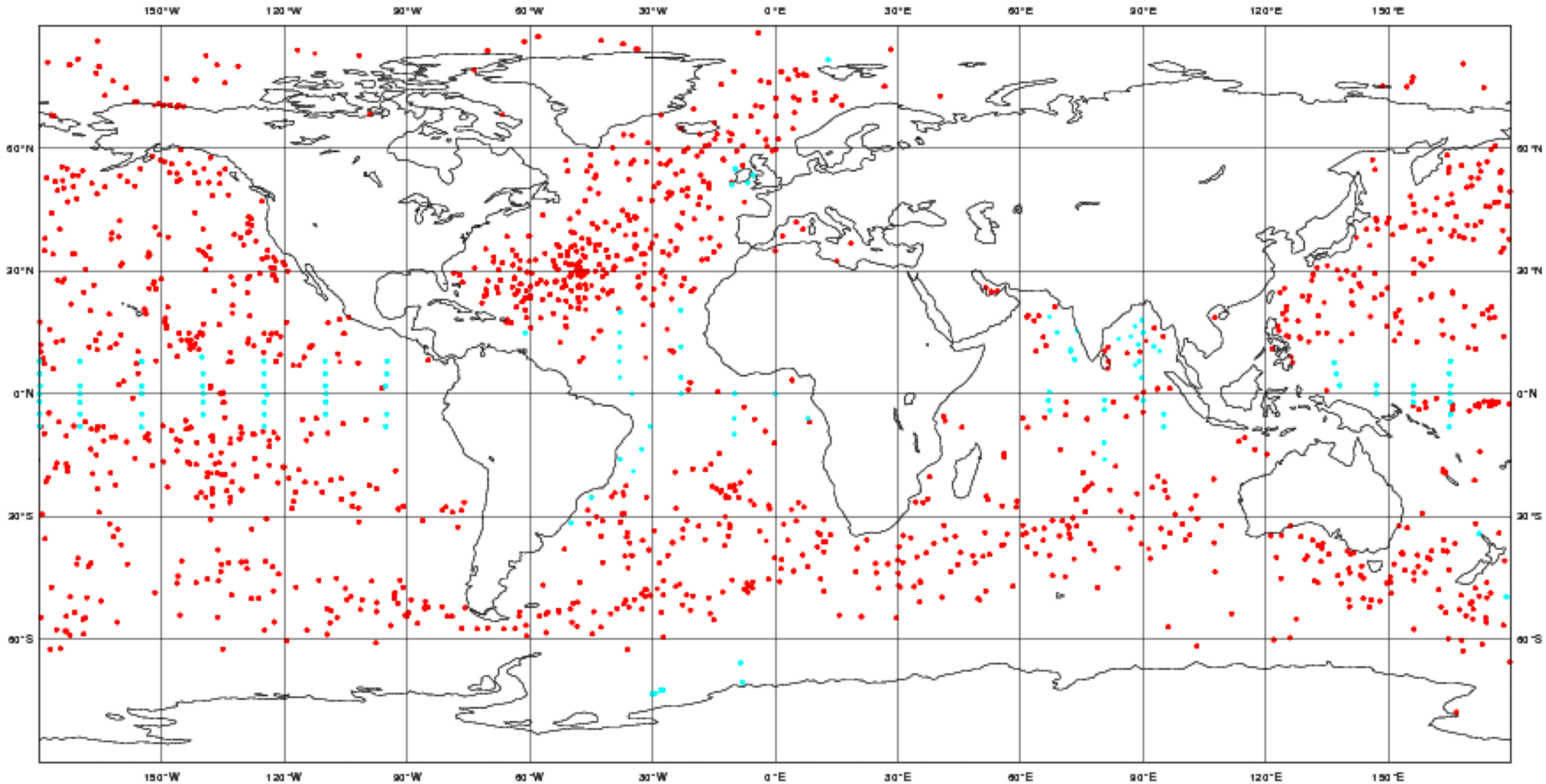
Magics 2.14.4 (64 bit)

ECMWF

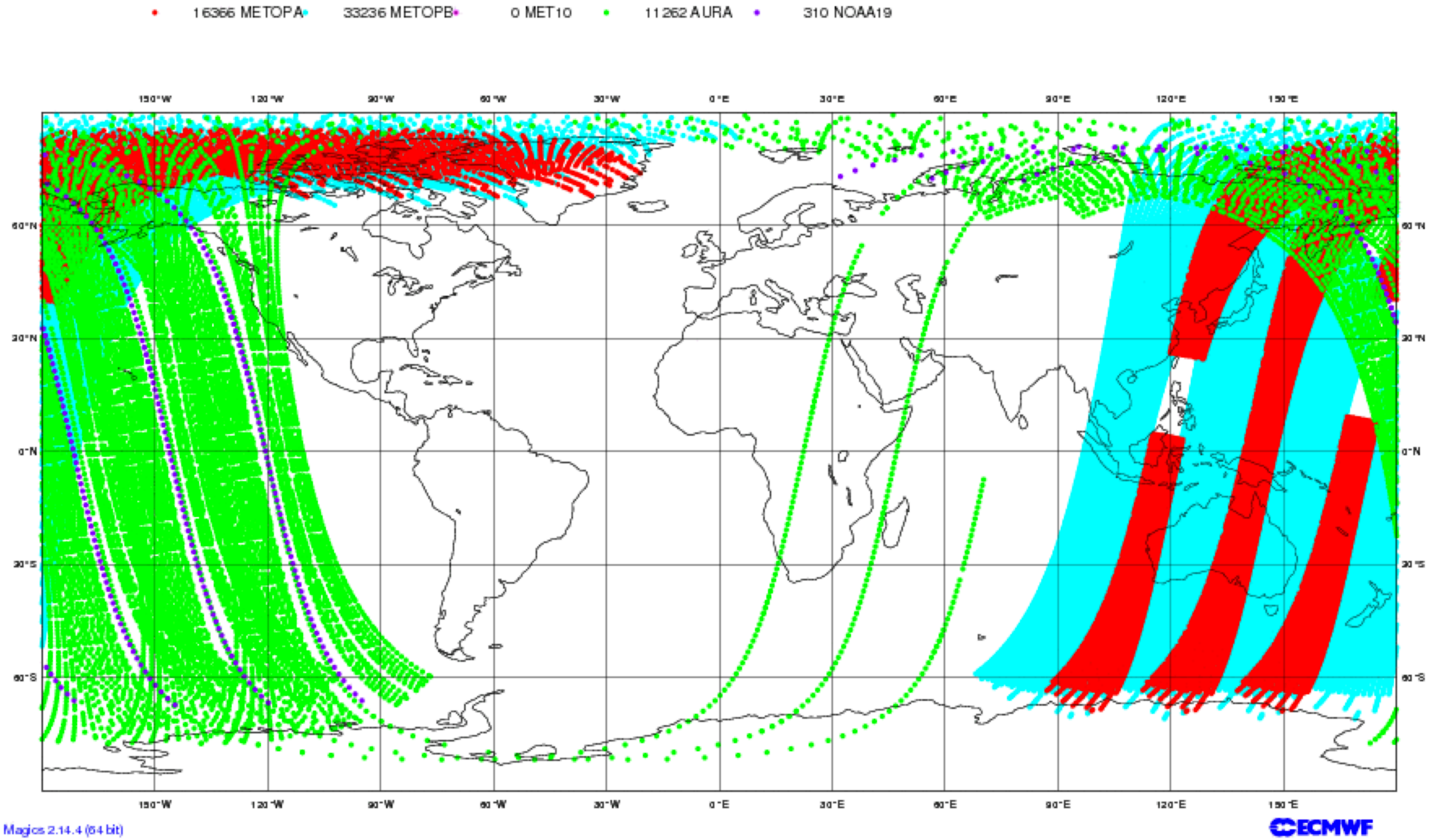
ECMWF Data Coverage (All obs DA) - Buoy
19/Apr/2015; 00 UTC
Total number of obs = 8533

8040 DRIFTER

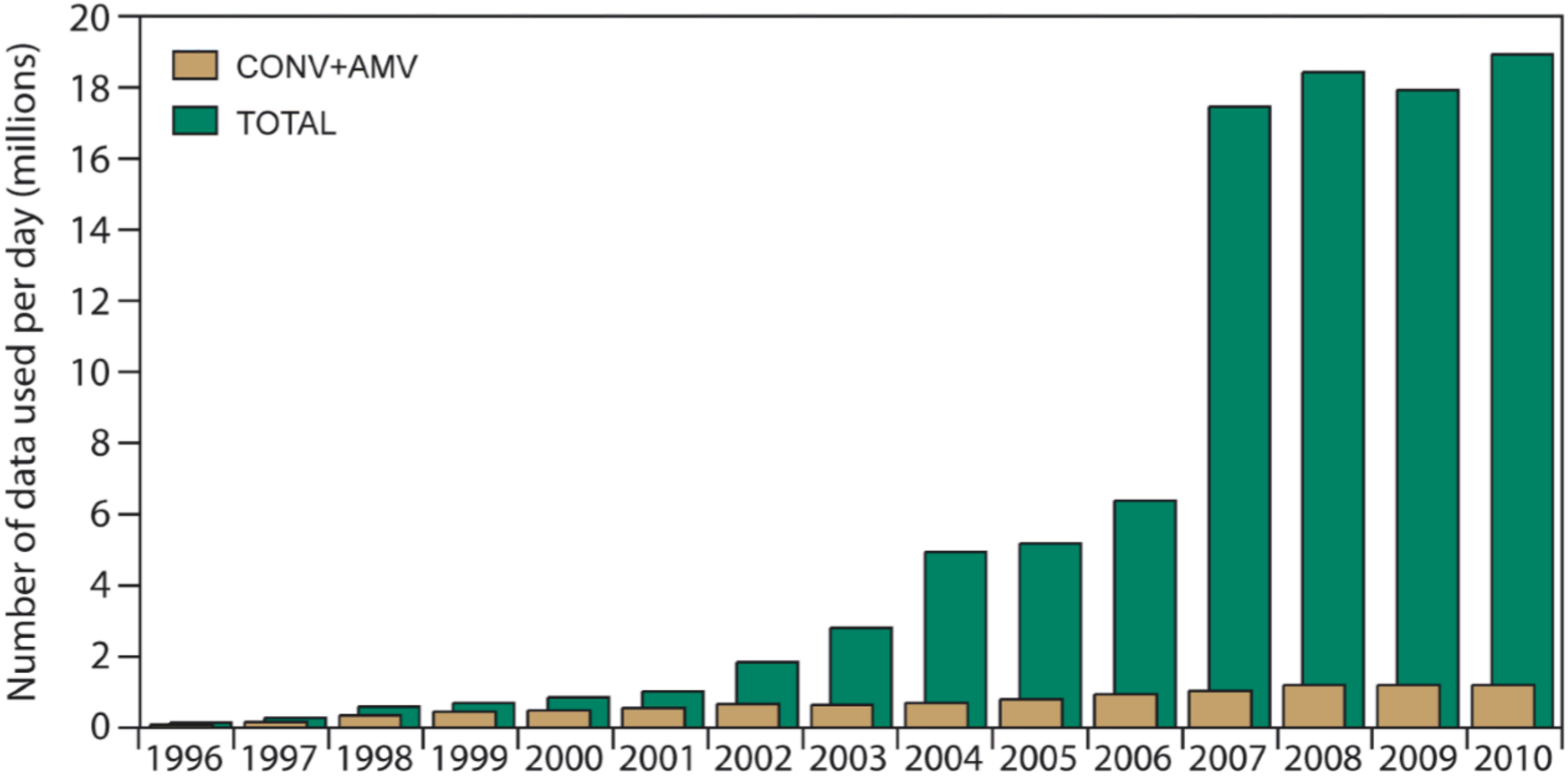
493 MOORED



ECMWF Data Coverage (All obs DA) - OZONE
19/Apr/2015; 00 UTC
Total number of obs = 61174



ECMWF



Value as of early 2013 : around 25 millions per day

- Observations *synoptiques* (observations au sol, radiosondages), effectuées simultanément, par convention internationale, dans toutes les stations météorologiques du globe (00:00, 06:00, 12:00, 18:00 UTC)

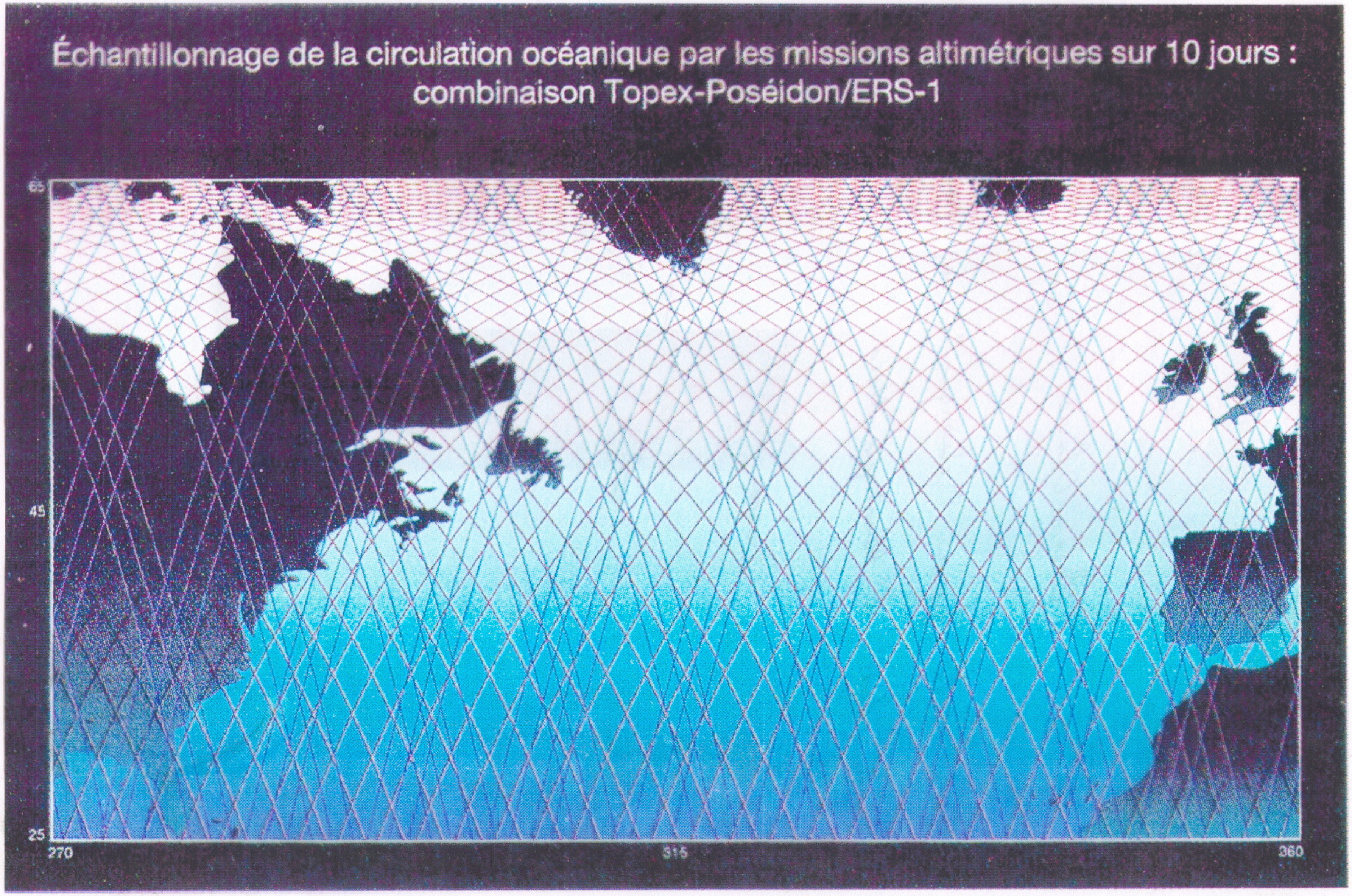
- Observations *asynoptiques* (satellites, avions), effectuées plus ou moins continûment dans le temps.

- Observations *directes* (température, pression, composantes du vent, humidité), portant sur les variables utilisées pour décrire l'état de l'écoulement dans les modèles numériques

- Observations *indirectes* (observations radiométriques, …), portant sur une combinaison plus ou moins complexe (le plus souvent, une intégrale d'espace unidimensionnelle) des variables utilisées pour décrire l'état de l'écoulement

$$y = H(\boldsymbol{x})$$

H : *opérateur d'observation* (par exemple, équation de transfert radiatif)



S. Louvel, Doctoral Dissertation, 1999

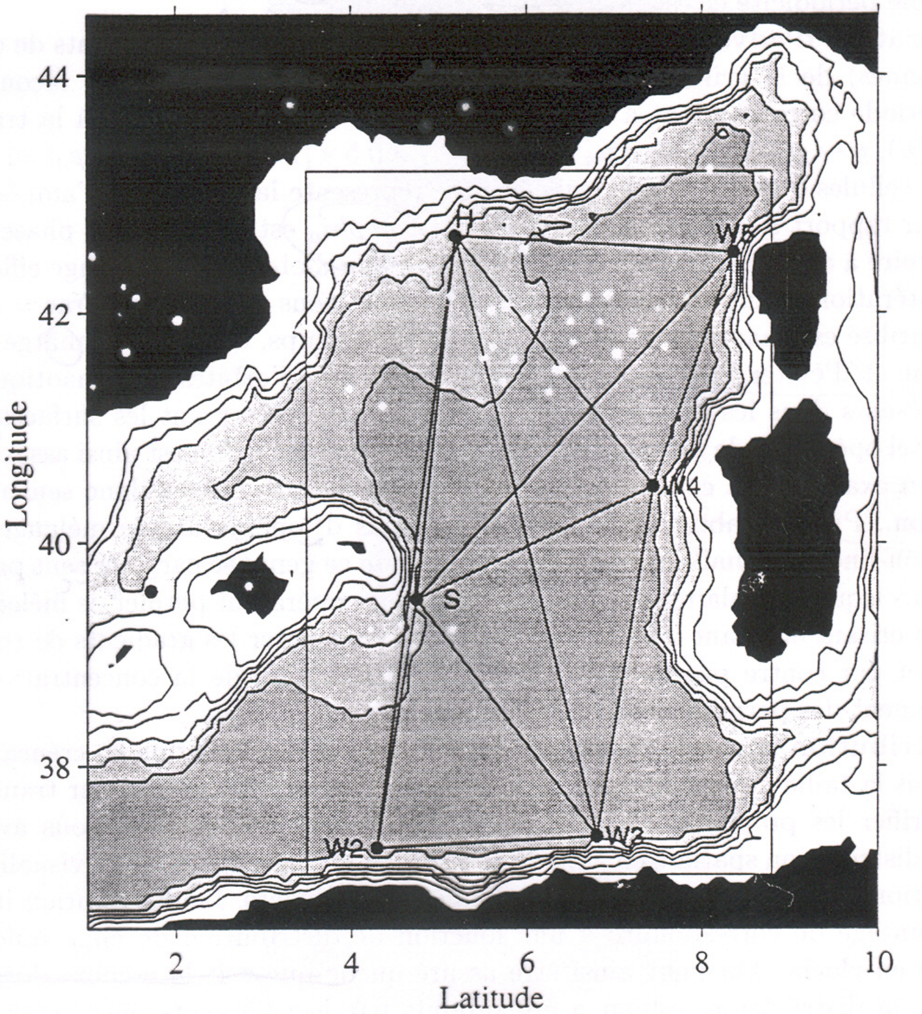


FIG. 1 – *Bassin méditerranéen occidental: réseau d'observation tomographique de l'expérience Thétis 2 et limites du domaine spatial utilisé pour les expériences numériques d'assimilation.*

E. Rémy, Doctoral Dissertation, 1999

Purpose of assimilation : reconstruct as accurately as possible the state of the atmospheric or oceanic flow, using all available appropriate information. The latter essentially consists of

- The observations proper, which vary in nature, resolution and accuracy, and are distributed more or less regularly in space and time.

- The physical laws governing the evolution of the flow, available in practice in the form of a discretized, and necessarily approximate, numerical model.

- 'Asymptotic' properties of the flow, such as, *e. g.*, geostrophic balance of middle latitudes. Although they basically are necessary consequences of the physical laws which govern the flow, these properties can usefully be explicitly introduced in the assimilation process.

Assimilation is one of many '*inverse problems*' encountered in many fields of science and technology

- solid Earth geophysics
- plasma physics
- 'nondestructive' probing
- navigation (spacecraft, aircraft, ….)
- …

Solution most often (if not always) based on Bayesian, or probabilistic, estimation. 'Equations' are fundamentally the same.

Difficulties specific to assimilation of meteorological observations :

- Very large numerical dimensions ($n \approx 10^6$-10^9 parameters to be estimated, $p \approx$ 1-3.10^7 observations per 24-hour period). Difficulty aggravated in Numerical Weather Prediction by the need for the forecast to be ready in time.

- Non-trivial, actually chaotic, underlying dynamics

Both observations and 'model' are affected with some uncertainty ⇒ uncertainty on the estimate.

For some reason, uncertainty is conveniently described by probability distributions (don't know too well why, but it works; see, *e.g.* Jaynes, 2007, *Probability Theory: The Logic of Science,* Cambridge University Press).

Assimilation is a problem in bayesian estimation.

Determine the conditional probability distribution for the state of the system, knowing everything we know (see Tarantola, A., 2005*, Inverse Problem Theory and Methods for Model Parameter Estimation*, SIAM).

Coût des différentes composantes de la chaîne de prévision opérationnelle du CEPMMT (septembre 2011, J.-N. Thépaut) :

4DVAR: 17%

EDA: 15%

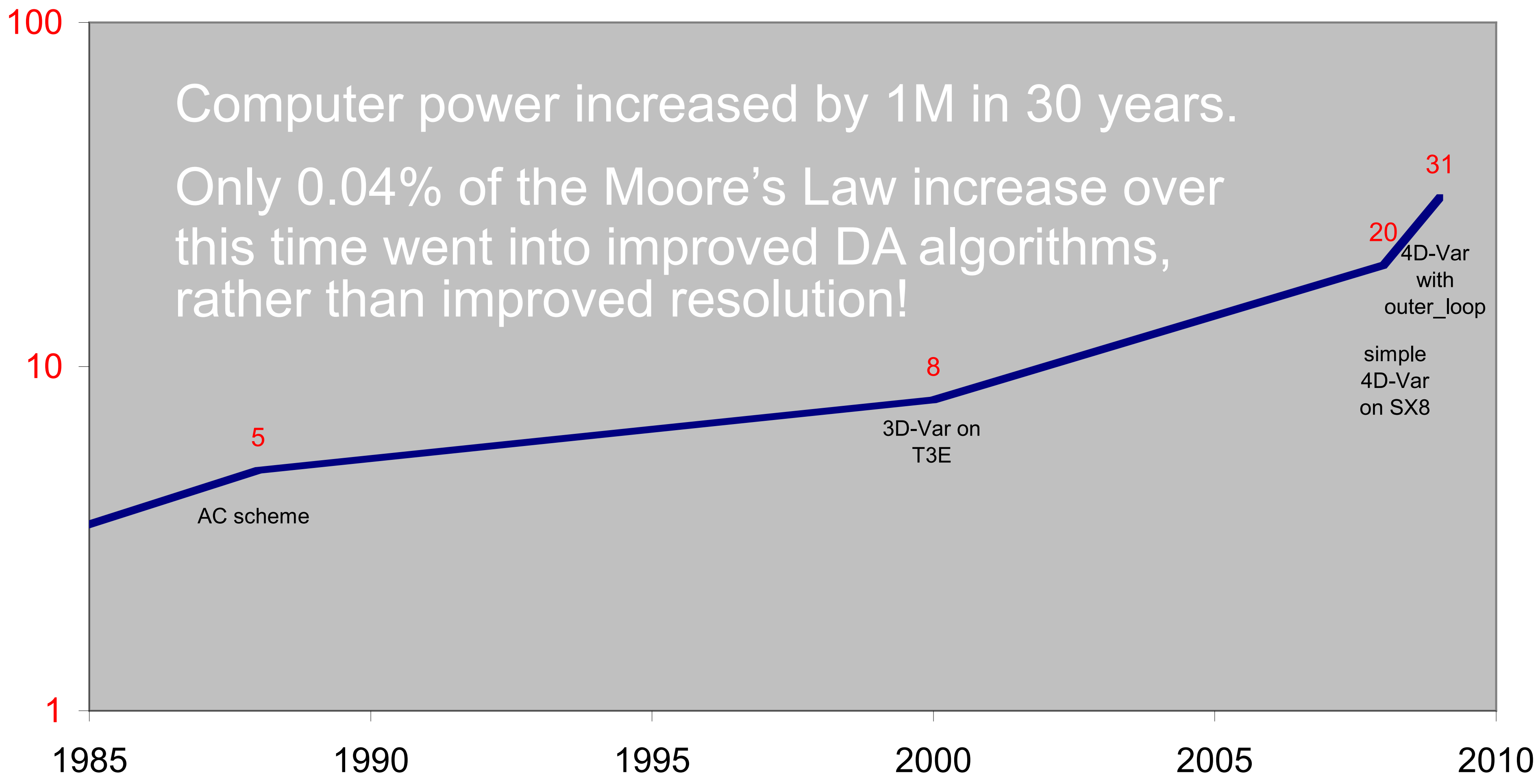
Modèle déterministe: 13%

EPS: 53%

autre: 2%

L'EDA fournit à la fois les variances d'erreur d'ébauche du 4D-Var, et les perturbations initiales (en complément des vecteurs singuliers) de l'EPS.

ratio of supercomputer costs: 1 day's assimilation / 1 day forecast



Courtesy A. Lorenc

Bayesian Estimation

Determine conditional probability distribution of the state of the system, given the probability distribution of the uncertainty on the data

$z_1 = x + \zeta_1$ $\qquad\qquad$ $\zeta_1 = \mathcal{N}[0, s_1]$

density function $p_1(\zeta) \propto \exp[\,-(\zeta^2)/2s_1]$

$z_2 = x + \zeta_2$ $\qquad\qquad$ $\zeta_2 = \mathcal{N}[0, s_2]$

density function $p_2(\zeta) \propto \exp[\,-(\zeta^2)/2s_2]$

- ζ_1 and ζ_2 mutually independent

What is the conditional probability $P(x = \xi \mid z_1, z_2)$ that x be equal to some value ξ ?

$z_1 = x + \zeta_1$ density function $p_1(\zeta) \propto \exp[-(\zeta^2)/2s_1]$

$z_2 = x + \zeta_2$ density function $p_2(\zeta) \propto \exp[-(\zeta^2)/2s_2]$

ζ_1 and ζ_2 mutually independent

$x = \xi \Leftrightarrow \zeta_1 = z_1 - \xi$ and $\zeta_2 = z_2 - \xi$

- $P(x = \xi \mid z_1, z_2) \propto p_1(z_1 - \xi)\, p_2(z_2 - \xi)$

$$\propto \exp[-(\xi - x^a)^2/2p^a]$$

where $1/p^a = 1/s_1 + 1/s_2$, $x^a = p^a (z_1/s_1 + z_2/s_2)$

Conditional probability distribution of x, given z_1 and z_2 : $\mathcal{N}[x^a, p^a]$

$p^a < (s_1, s_2)$ independent of z_1 and z_2

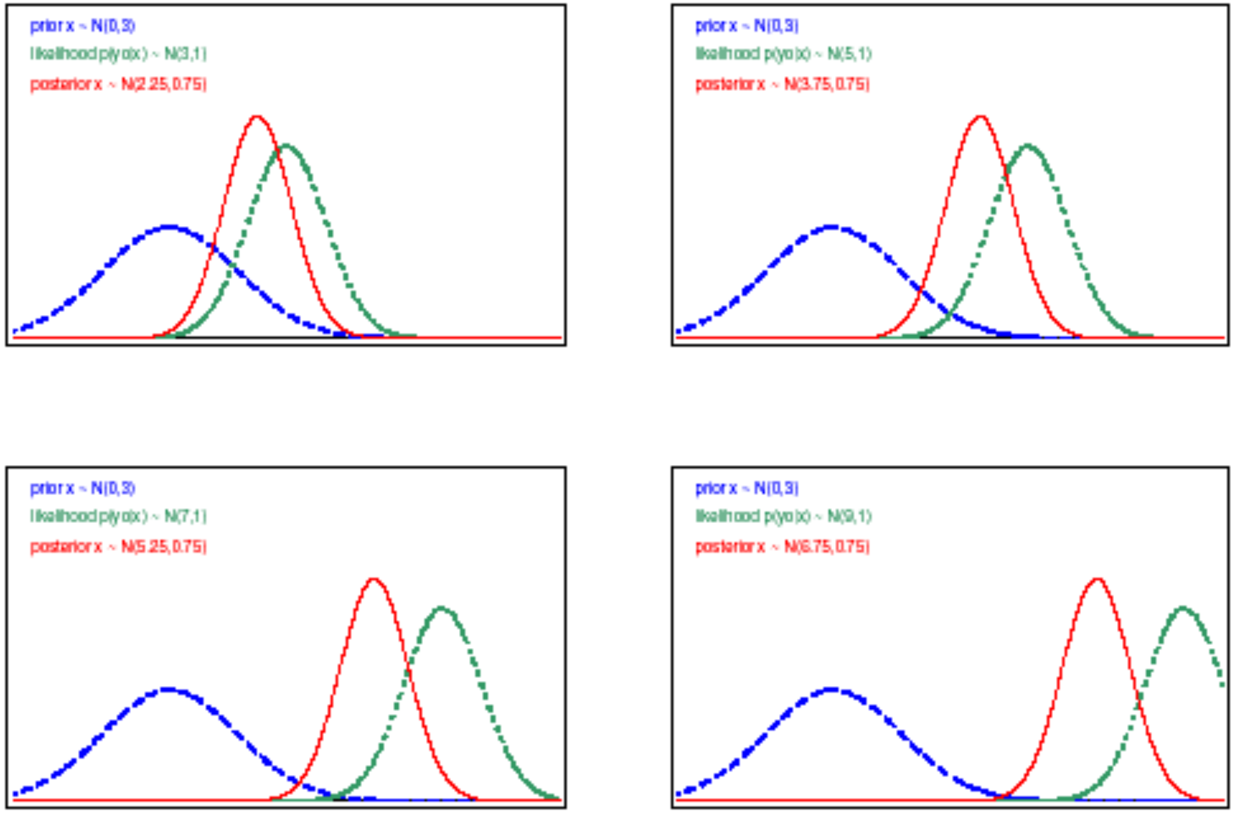


Fig. 1.1: Prior pdf $p(x)$ (dashed line), posterior pdf $p(x|y^o)$ (solid line), and Gaussian likelihood of observation $p(y^o|x)$ (dotted line), plotted against x for various values of y^o. (Adapted from Lorenc and Hammon 1988.)

Conditional expectation x^a minimizes following scalar *objective function*, defined on ξ-space

$$\xi \rightarrow \mathcal{J}(\xi) \equiv (1/2) [(z_1 - \xi)^2 / s_1 + [(z_2 - \xi)^2 / s_2]$$

In addition

$$p^a = 1/ \mathcal{J}''(\xi)$$

Conditional probability distribution in Gaussian case

$$P(x = \xi \mid z_1, z_2) \propto \exp[- \underbrace{(\xi - x^a)^2/2p^a}_{\mathcal{J}(\xi) + Cst}]$$

Estimate

$$x^a = p^a \, (z_1/s_1 + z_2/s_2)$$

with error p^a such that

$$1/p^a = 1/s_1 + 1/s_2$$

can be obtained, independently of any Gaussian hypothesis, as simply corresponding to the linear combination of z_1 and z_2 that minimizes the error $E\,[(x^a - x)^2]$

Best Linear Unbiased Estimator (BLUE)

$z_1 = x + \zeta_1$
$z_2 = x + \zeta_2$

Same as before, but ζ_1 and ζ_2 are now distributed according to exponential law with parameter a, *i. e.*

$$p(\zeta) \propto \exp[-|\zeta|/a] \quad ; \quad \mathrm{Var}(\zeta) = 2a^2$$

Conditional probability density function is now uniform over interval $[z_1, z_2]$, exponential with parameter $a/2$ outside that interval

$$E(x \mid z_1, z_2) = (z_1+z_2)/2$$

$\mathrm{Var}(x \mid z_1, z_2) = a^2\,(2\delta^3/3 + \delta^2 + \delta + 1/2) / (1 + 2\delta)$, with $\delta = |z_1 - z_2|/(2a)$

Increases from $a^2/2$ to ∞ as δ increases from 0 to ∞. Can be larger than variance $2a^2$ of original errors (probability 0.08)

~~(Entropy $-\int p \ln p$ always decreases in bayesian estimation)~~

Bayesian estimation

State vector x, belonging to *state space* $\mathcal{S}$ ($\dim \mathcal{S} = n$), to be estimated.

Data vector z, belonging to *data space* $\mathcal{D}$ ($\dim \mathcal{D} = m$), available.

$$z = F(x, \zeta) \qquad (1)$$

where ζ is a random element representing the uncertainty on the data (or, more precisely, on the link between the data and the unknown state vector).

For example

$$z = \Gamma x + \zeta$$

Bayesian estimation (continued)

Probability that $x = \xi$ for given ξ ?

$$x = \xi \quad \Rightarrow \quad z = F(\xi, \zeta)$$

$$P(x = \xi \mid z) = P[z = F(\xi, \zeta)] \,/ \int_{\xi'} P[z = F(\xi', \zeta)]$$

Unambiguously defined iff, for any ζ, there is at most one x such that (1) is verified.

$\Leftrightarrow$ data contain information, either directly or indirectly, on any component of x. *Determinacy* condition.

Bayesian estimation is however impossible in its general theoretical form in meteorological or oceanographical practice because

- It is impossible to explicitly describe a probability distribution in a space with dimension even as low as $n \approx 10^3$, not to speak of the dimension $n \approx 10^{6\text{-}9}$ of present Numerical Weather Prediction models.

- Probability distribution of errors on data very poorly known (model errors in particular).

One has to restrict oneself to a much more modest goal. Two approaches exist at present

- Obtain some 'central' estimate of the conditional probability distribution (expectation, mode, …), plus some estimate of the corresponding spread (standard deviations and a number of correlations).

- Produce an ensemble of estimates which are meant to sample the conditional probability distribution (dimension $N \approx O(10\text{-}100)$).

Random vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^T = (x_i)$ (*e. g.* pressure, temperature, abundance of given chemical compound at n grid-points of a numerical model)

- Expectation $E(\boldsymbol{x}) \equiv [E(x_i)]$; centred vector $\boldsymbol{x}' \equiv \boldsymbol{x} - E(\boldsymbol{x})$

- Covariance matrix

$$E(\boldsymbol{x}'\boldsymbol{x}'^T) = [E(x_i' x_j')]$$

dimension nxn, symmetric non-negative (strictly definite positive except if linear relationship holds between the x_i''s with probability 1).

- Two random vectors

$\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^T$

$\boldsymbol{y} = (y_1, y_2, \ldots, y_p)^T$

$$E(\boldsymbol{x}'\boldsymbol{y}'^T) = E(x_i' y_j')$$

dimension nxp

Covariance matrices will be denoted

$$C_{\boldsymbol{xx}} \equiv E(\boldsymbol{x}'\boldsymbol{x}'^{\mathrm{T}})$$

$$C_{\boldsymbol{xy}} \equiv E(\boldsymbol{x}'\boldsymbol{y}'^{\mathrm{T}})$$

Random function $\Phi(\xi)$ (field of pressure, temperature, abundance of given chemical compound, … ; ξ is now spatial and/or temporal coordinate)

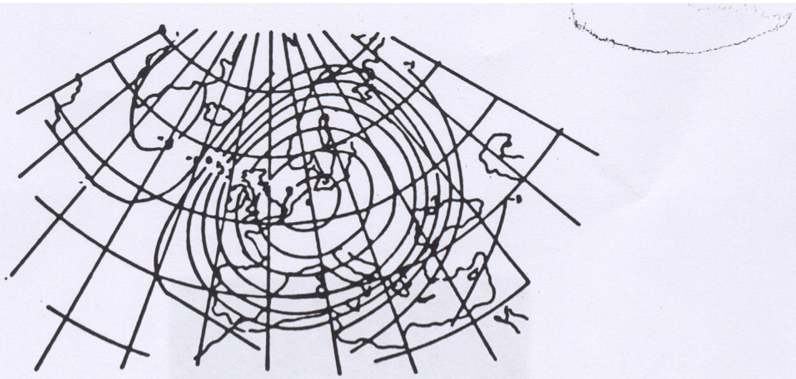
- Expectation $E[\Phi(\xi)]$; $\Phi'(\xi) \equiv \Phi(\xi) - E[\Phi(\xi)]$
- Variance $Var[\Phi(\xi)] = E\{[\Phi'(\xi)]^2\}$

- Covariance function

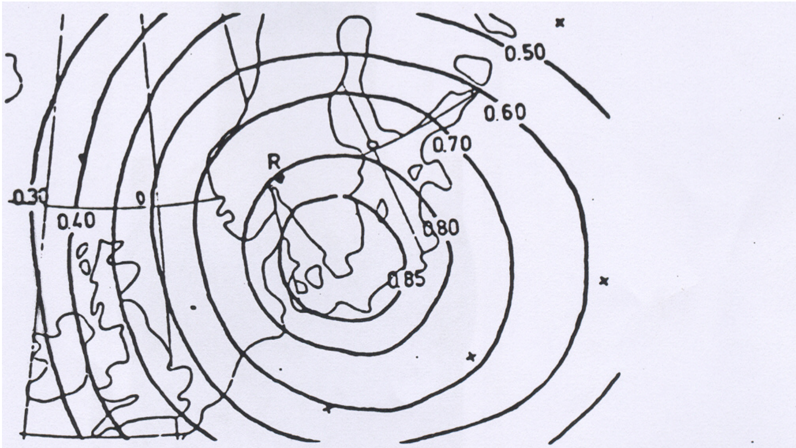
$$(\xi_1, \xi_2) \rightarrow C_\Phi(\xi_1, \xi_2) \equiv E[\Phi'(\xi_1)\ \Phi'(\xi_2)]$$

- Correlation function

$$Cor_\Phi(\xi_1, \xi_2) \equiv E[\Phi'(\xi_1)\ \Phi'(\xi_2)] / \{Var[\Phi(\xi_1)]\ Var[\Phi(\xi_2)]\}^{1/2}$$



.: Isolines for the auto-correlations of the 500 mb geopotential between the station in Hannover and surrounding stations.
From Bertoni and Lund (1963)



: Isolines of the cross-correlation between the 500 mb geopotential in station 01 384 (R) and the surface pressure in surrounding stations.

After N. Gustafsson

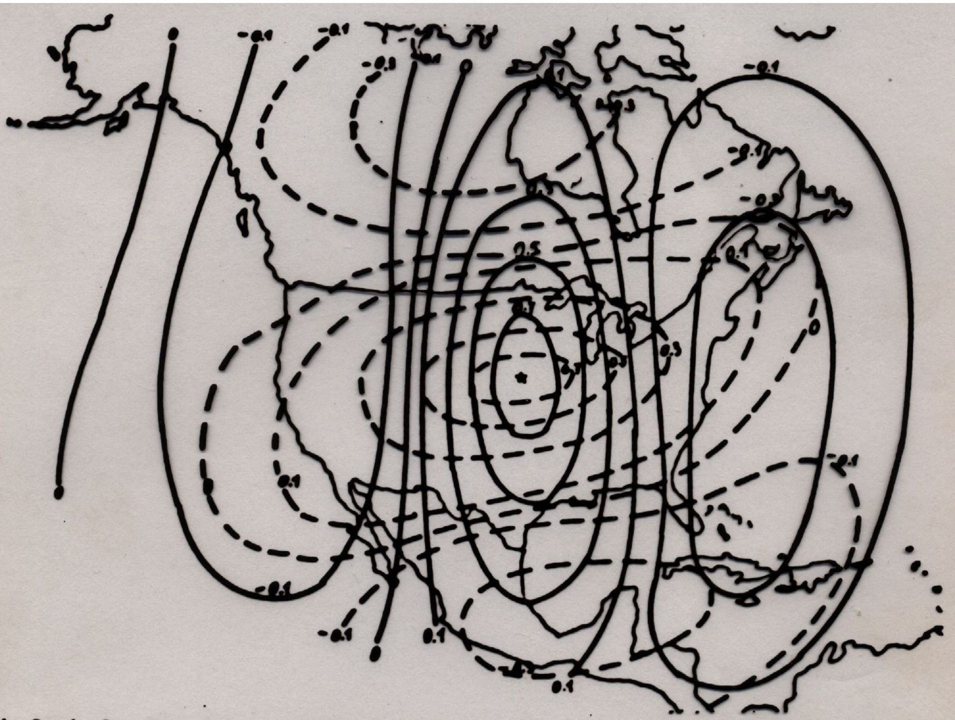


Figure 4.2.4.3: Isolines for the auto-correlation of the 500 mb u-wind component (dashed line) and the auto-correlation of the 500 mb v-wind component (full line). The "star" indicates the position of the reference station. (From Buel (1972).

After N. Gustafsson

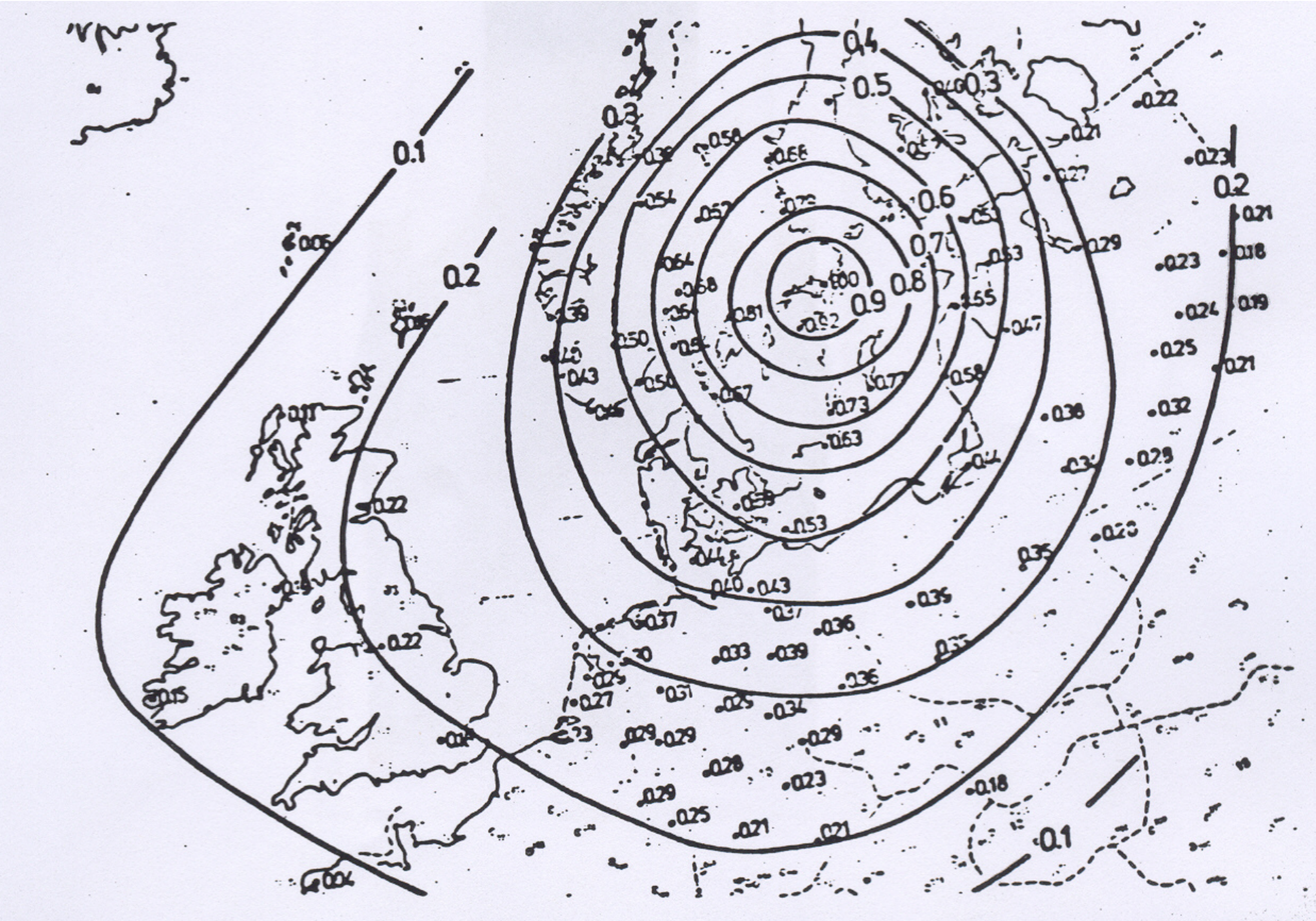


Figure 5.1.1.4.1 Auto-correlation of errors in 12h numerical forecasts of surface pressure in a reference station (Stockholm) and other stations.

After N. Gustafsson