École Doctorale des Sciences de l'Environnement d’Île-de-France

Année Universitaire 2016-2017

Modélisation Numérique de l’Écoulement Atmosphérique et Assimilation de Données

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Cours 1

6 Avril 2017

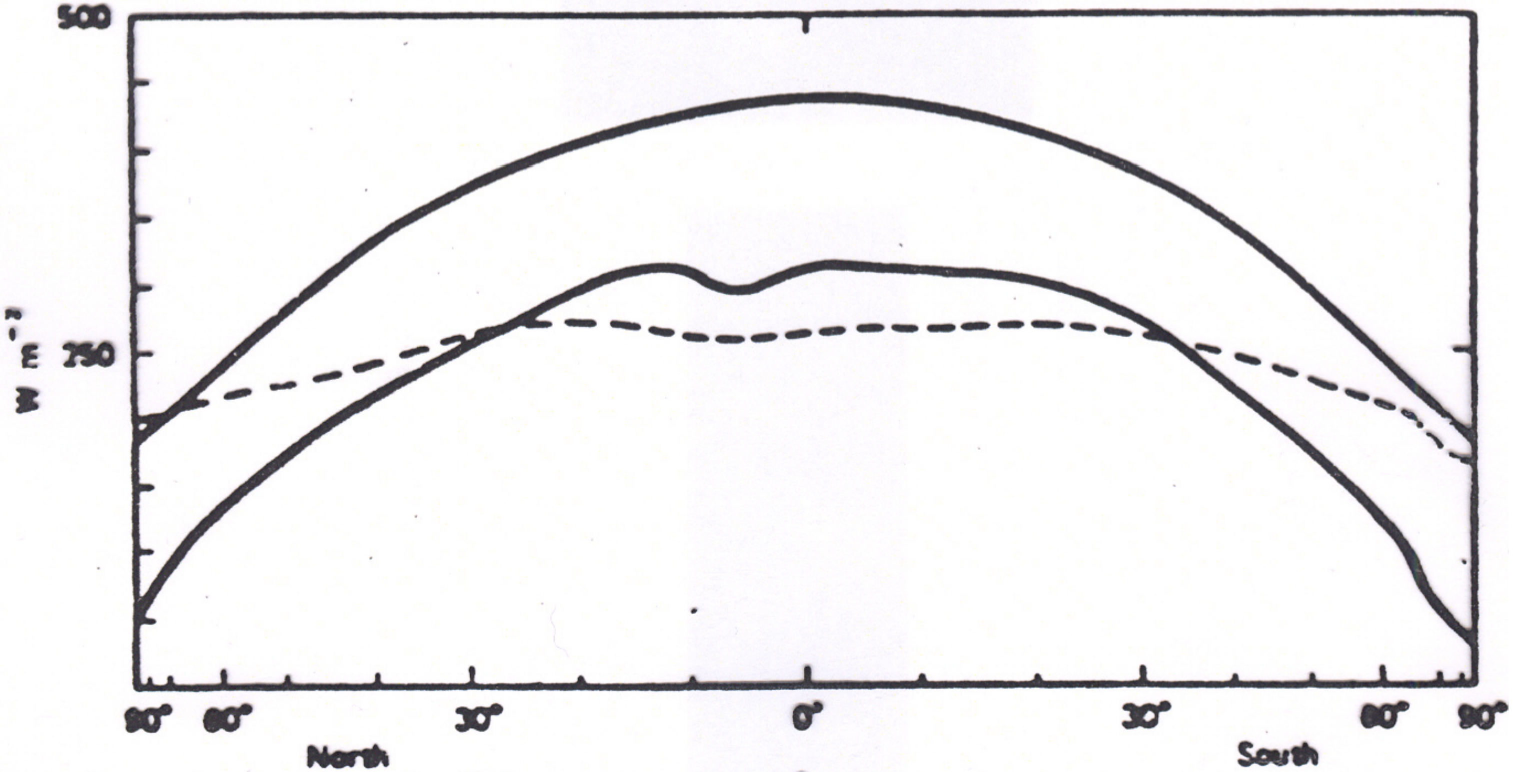
Programme of the course

1. Numerical modeling of the atmospheric flow. The *primitive* equations. Discretization methods. Numerical Weather Prediction. Present performance.
2. The meteorological observation system. The problem of 'assimilation'. Bayesian estimation. Random variables and random functions. Meteorological examples.
3. 'Optimal Interpolation'. Basic properties. Meteorological applications. The theory of *Best Linear Unbiased Estimator*.
4. Advanced assimilation methods.

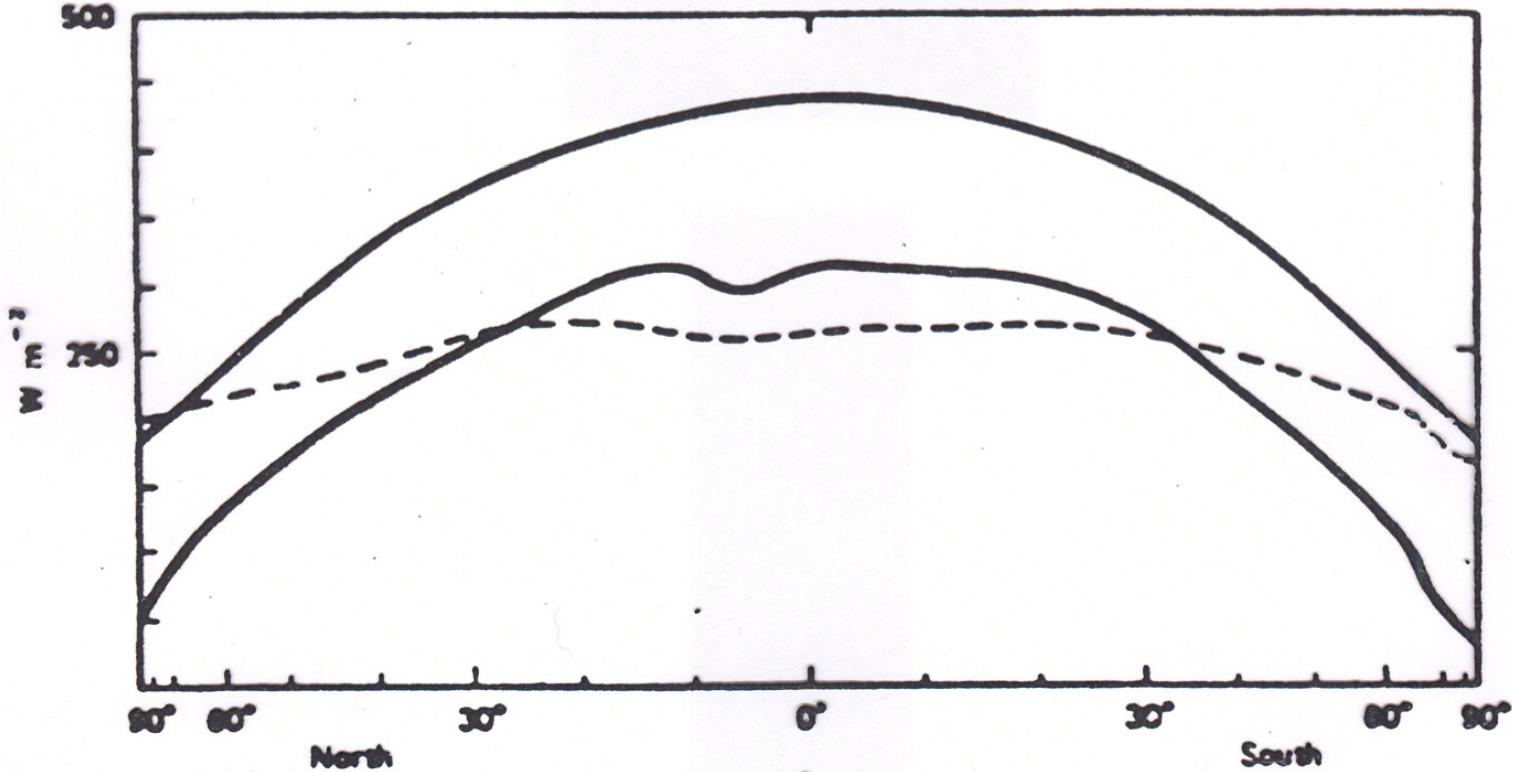
   - Kalman Filter. Ensemble Kalman Filter. Present performance and perspectives.

   - Variational Assimilation. Adjoint Equations. Present performance and perspectives.
5. Advanced assimilation methods (continuation).

   - Bayesian Filters. Theory, present performance and perspectives.

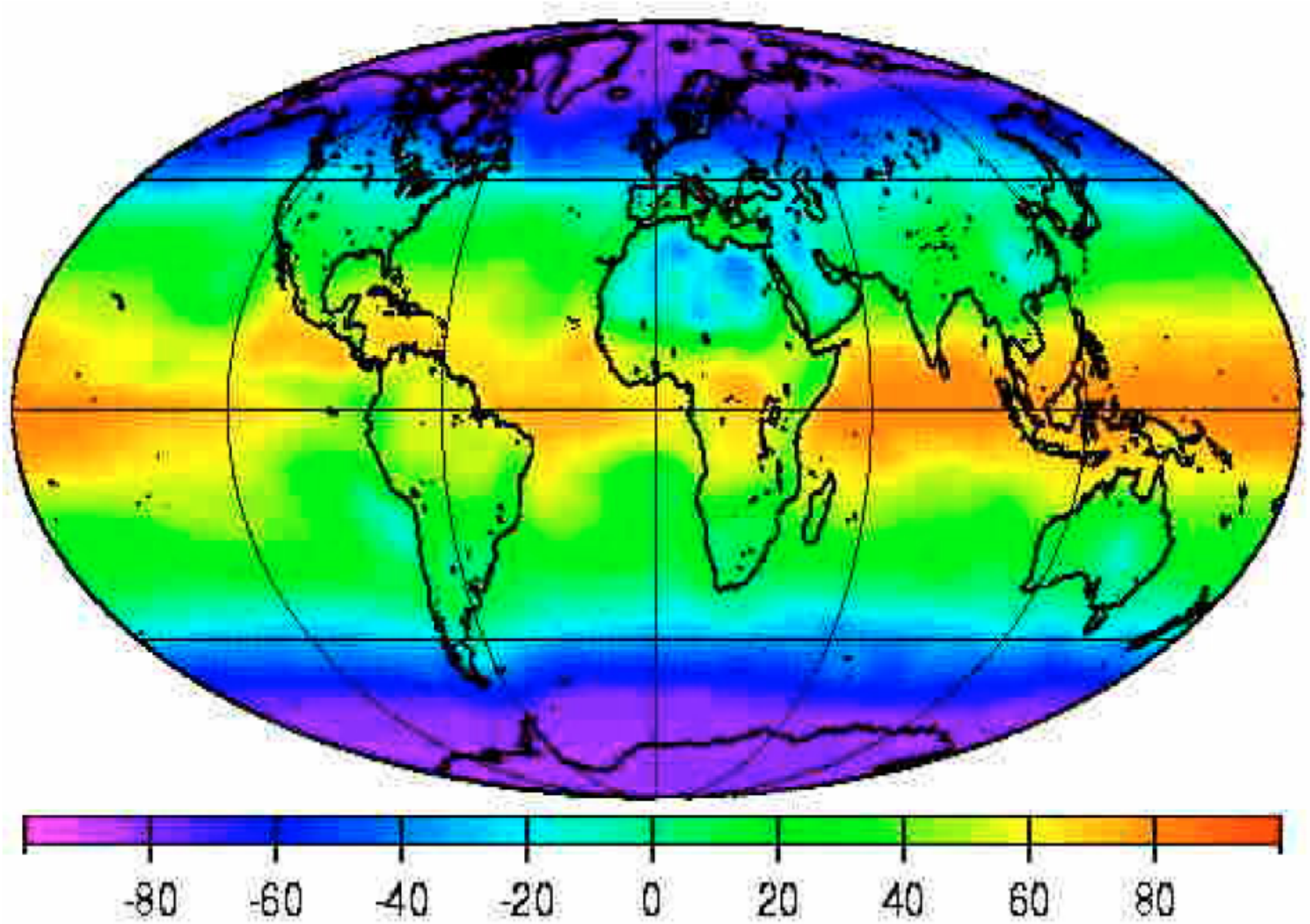


500
250
W m⁻²
90°
60°
30°
0°
30°
60°
90°
North
South



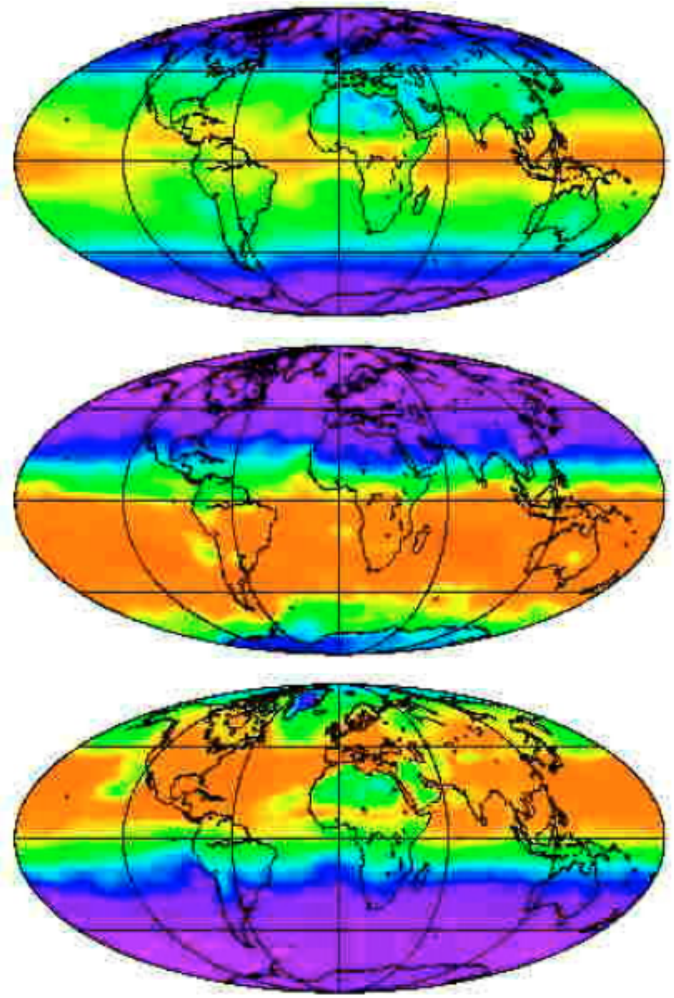
Bilan radiatif de la Terre, moyenné sur un an

Bilan radiatif au sommet de l'atmosphère (en W m^{-2})

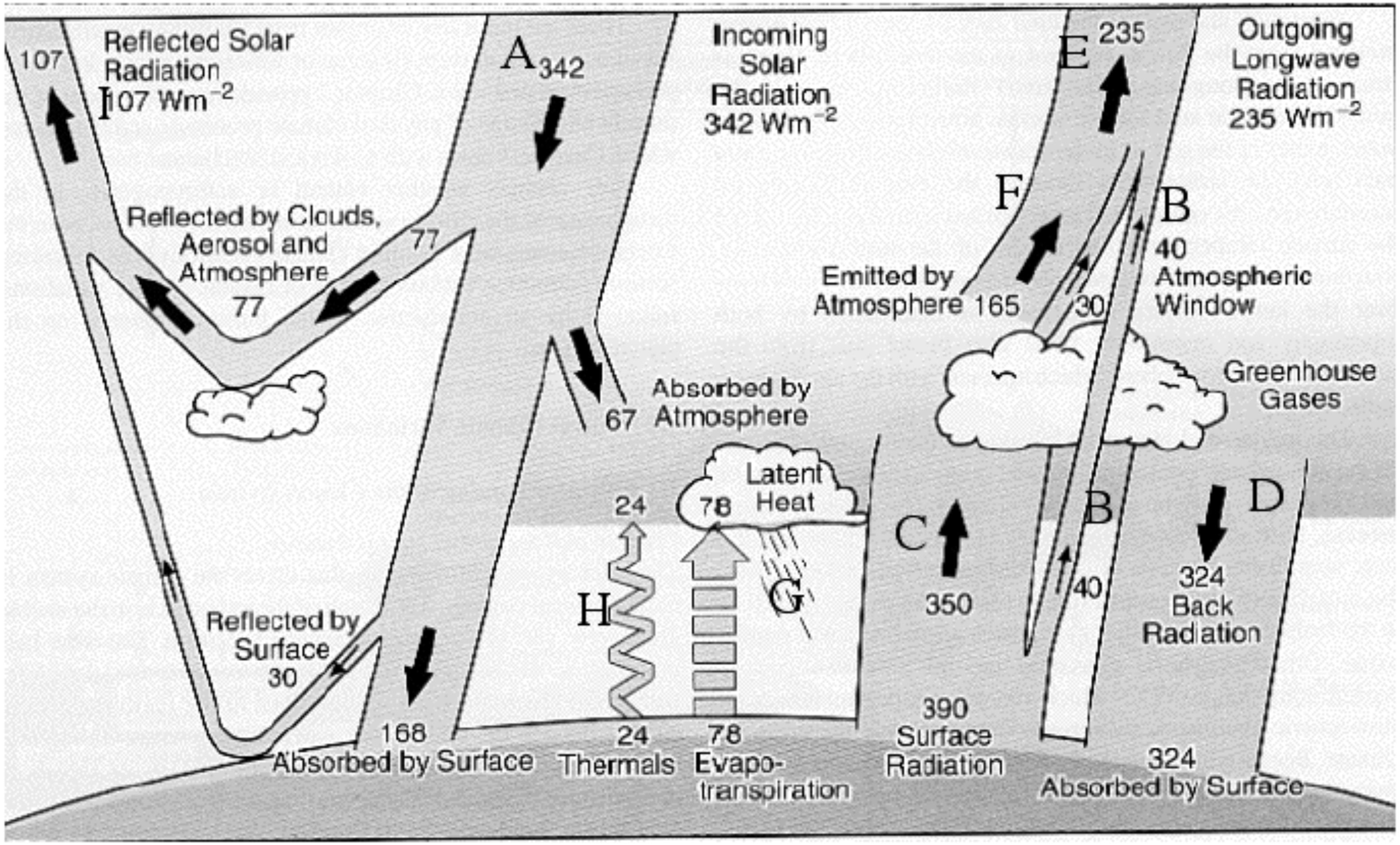


Cours H. Le Treut

Variations saisonnières du bilan radiatif



Cours H. Le Treut



D’après K. Trenberth

Particle moves on sphere with radius R
under the action of a force lying
in meridian plane of the particle

→ Angular momentum wrt axis of rotation conserved.

$$(u + \Omega R \cos\varphi) R \cos\varphi = \mathrm{Cst}$$

On Earth, $\Omega \approx 2\pi\, 10^{-5}\ s^{-1}$, $R \approx 6.4\ 10^{6}\ m$.

If $u = 0$ at equator, $u = 329\ ms^{-1}$ at latitude $\varphi = 45°$. If $u = 0$ at $45°$, $u = -232\ ms^{-1}$ at equator.

Hadley, G., 1735, Concerning the cause of the general trade winds, *Philosophical Transactions of the Royal Society*

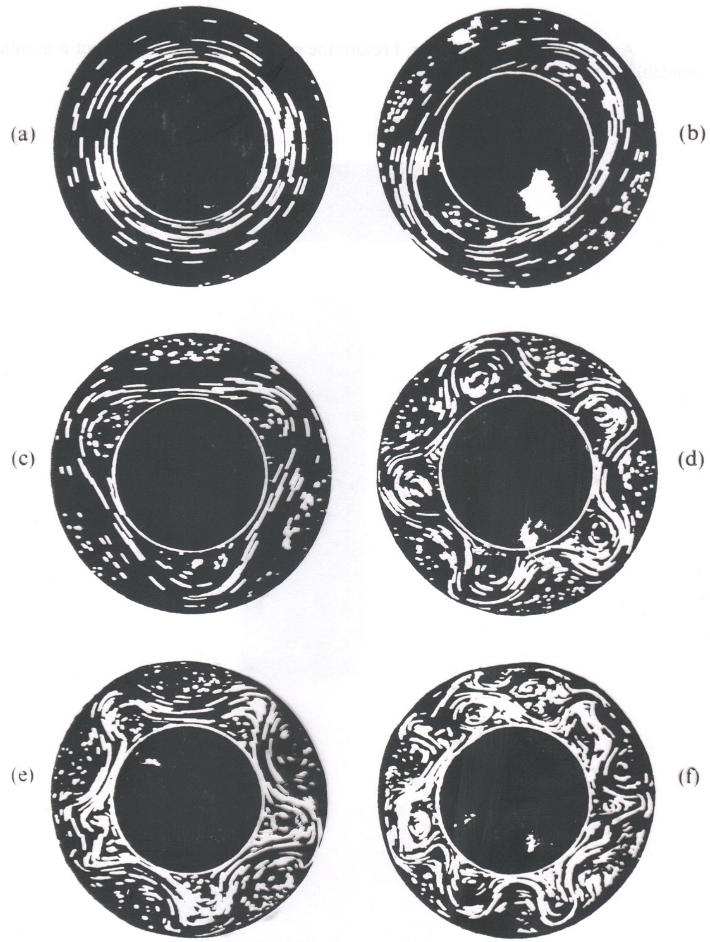


Fig. 10.1. Streak photographs illustrating the dependence of the flow type on rotation rate Ω for a laboratory 'dishpan' experiment. The values of Ω in rad s^{-1} are (a) 0.41; (b) 1.07; (c) 1.21; (d) 3.22; (e) 3.91; (f) 6.4. Working fluid was a water–glycerol solution of mean density 1.037 g cm^{-3} and kinematic viscosity 1.56×10^{-2} cm^2 s^{-1}. The streak photographs show the flow at a depth of 0.5 cm below the free upper surface (see also problem 10.1.) (From Hide & Mason, 1975)

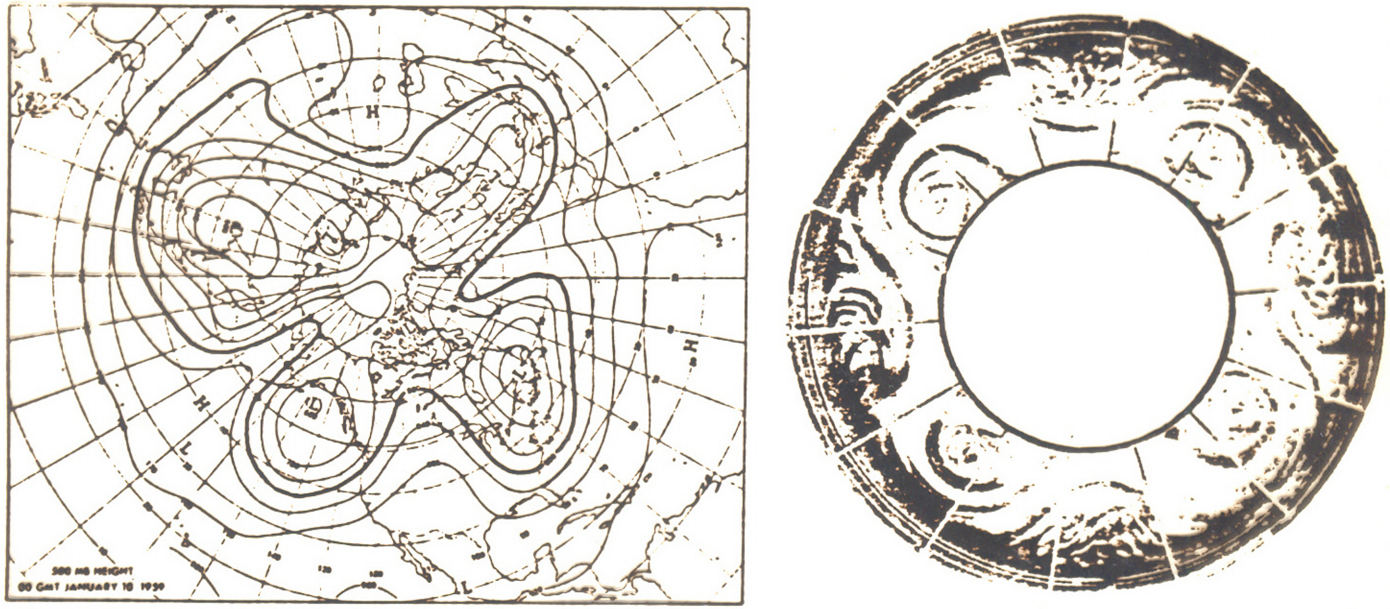
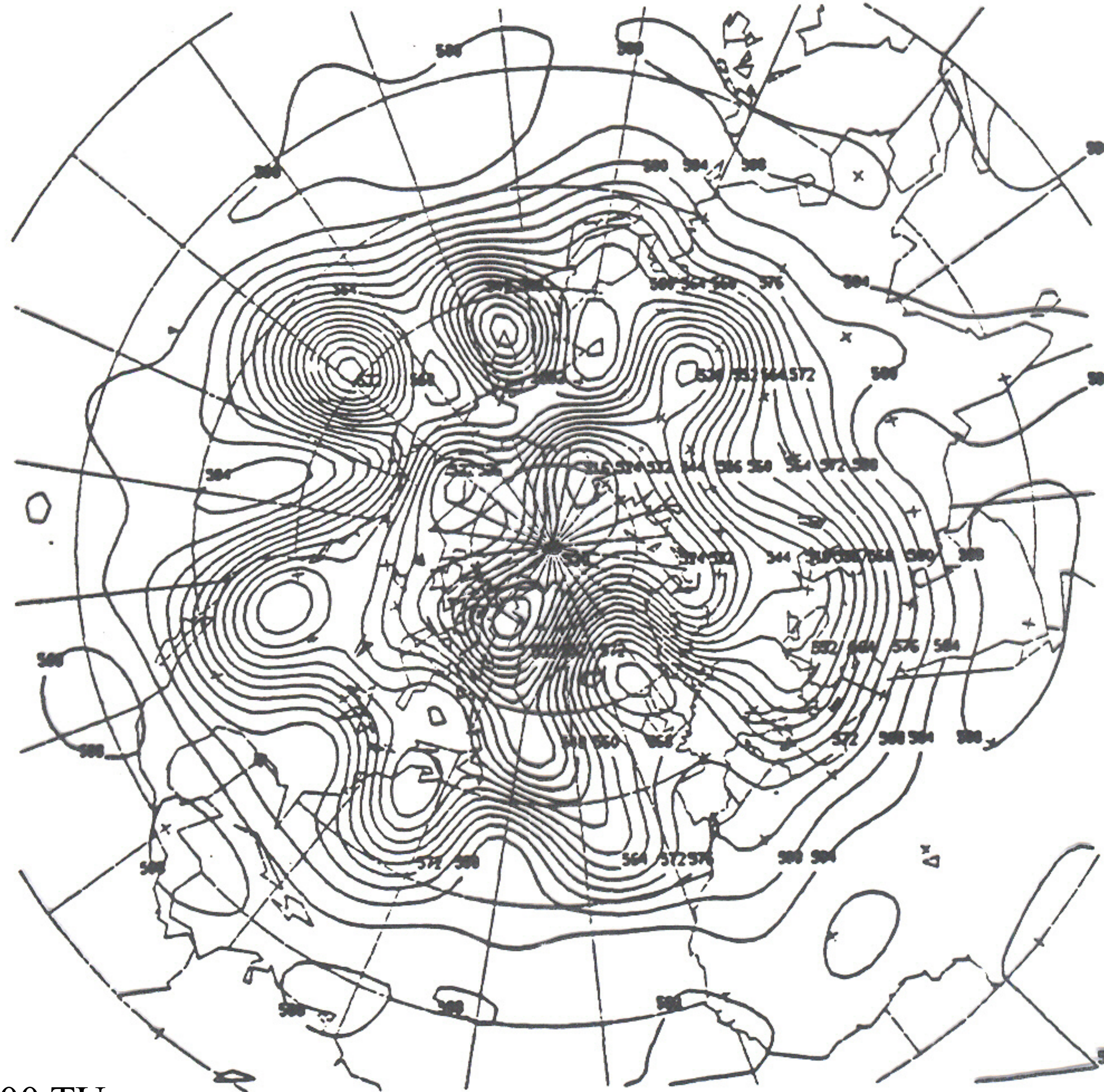


Figure 2. Comparison shows similarities between the global 500 mb pressure pattern in the upper atmosphere of the Northern Hemisphere and a four-wave pattern in the laboratory. (Laboratory flow conditions were similar to those in Fig. 1, except $\Omega = 1.95$ radians per sec.) In the atmosphere the flow is approximately parallel to the isobars (the flow is to the right, from high to low pressure), with speed inversely proportional to the spacing. Changes in the wave pattern have a significant effect on large-scale weather and climate.



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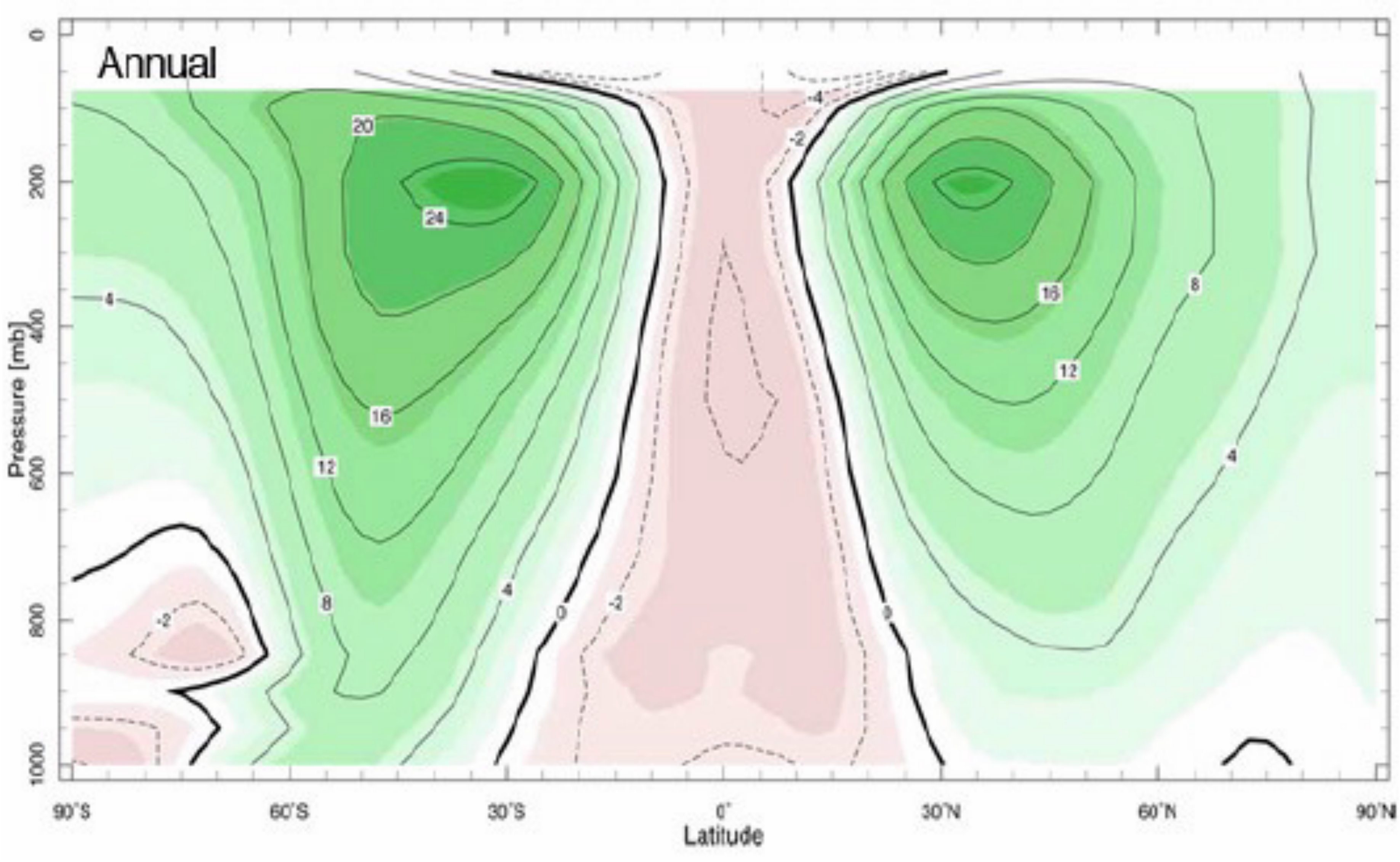
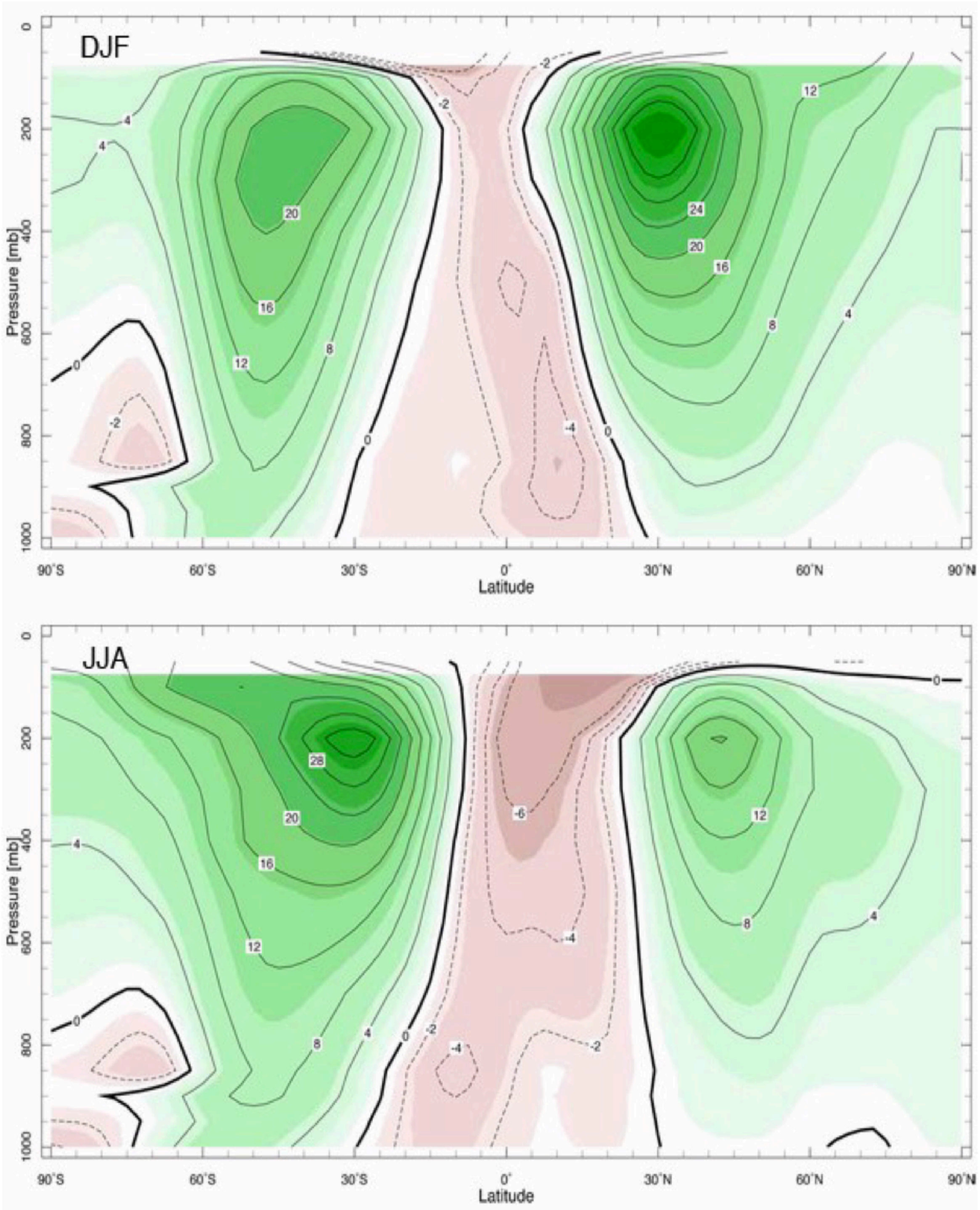
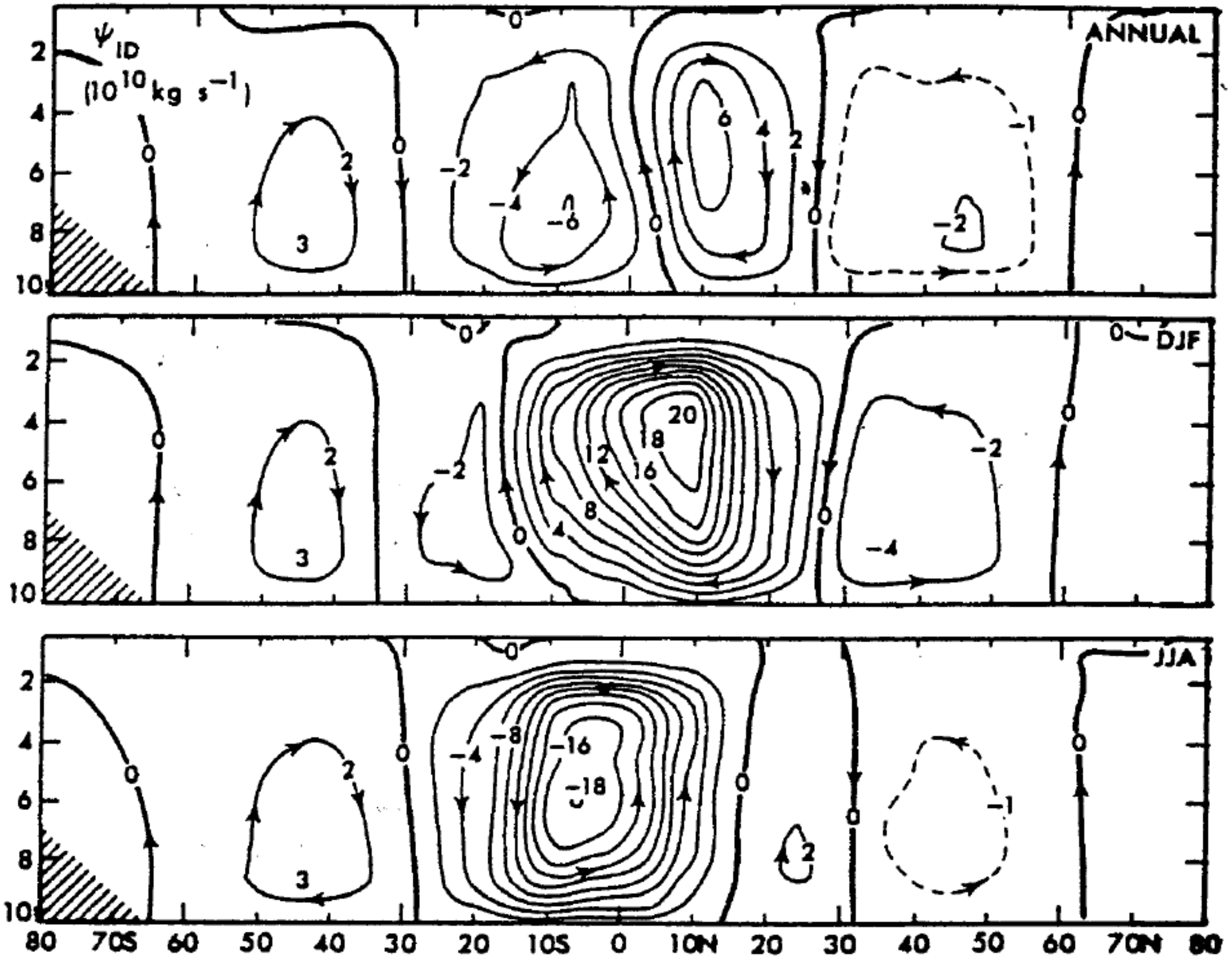


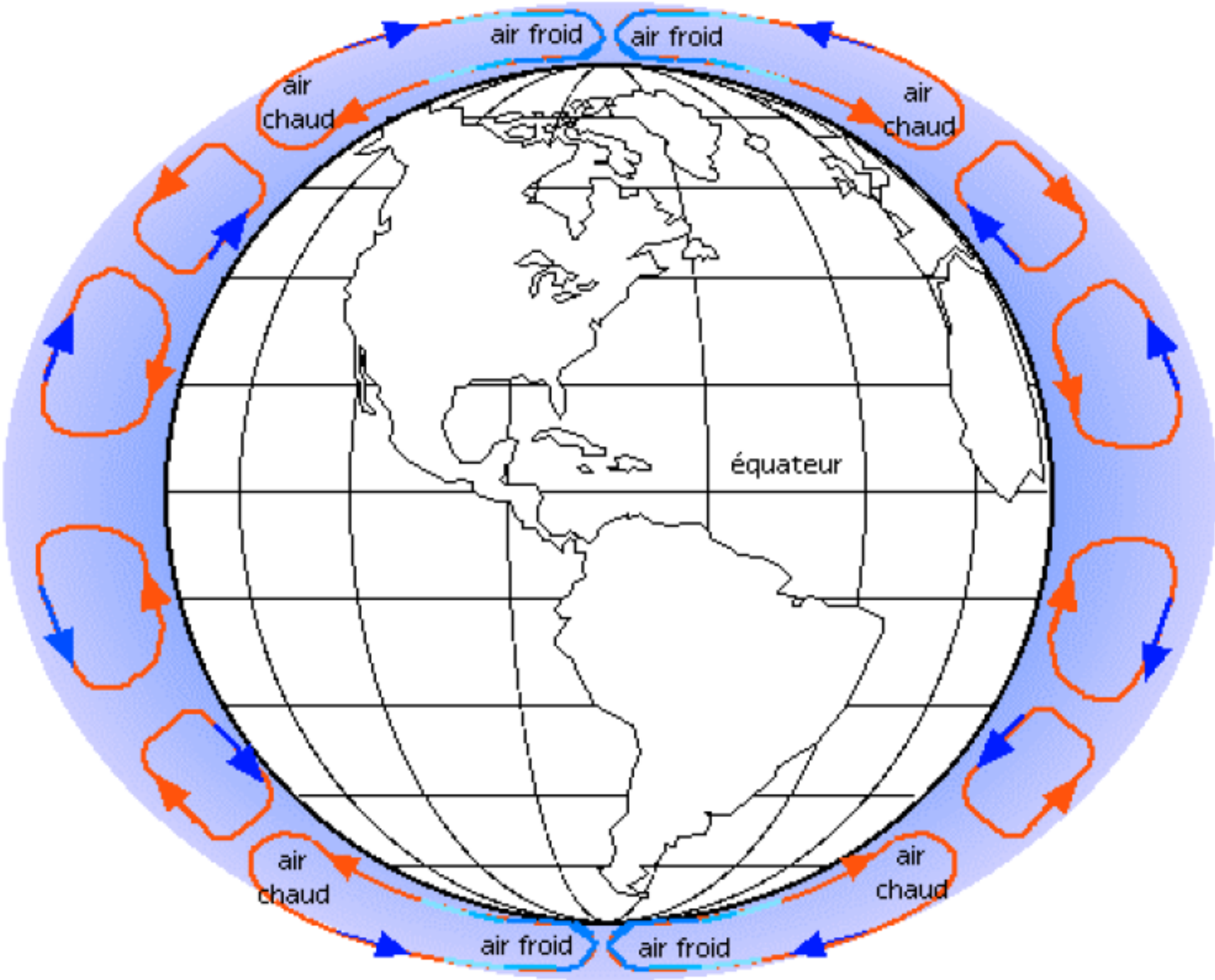
Vent zonal; moyenne longitudinale annuelle ($m.s^{-1}$)
http://paoc.mit.edu/labweb/notes/chap5.pdf,
Atmosphere, Ocean and Climate Dynamics, by J. Marshall and R. A. Plumb,
International Geophysics, Elsevier)



Vent zonal; moyenne longitudinale saisonnière ($m.s^{-1}$, *ibid*.)



Peixoto and Oort, 1992, *The Physics of Climate,* Springer-Verlag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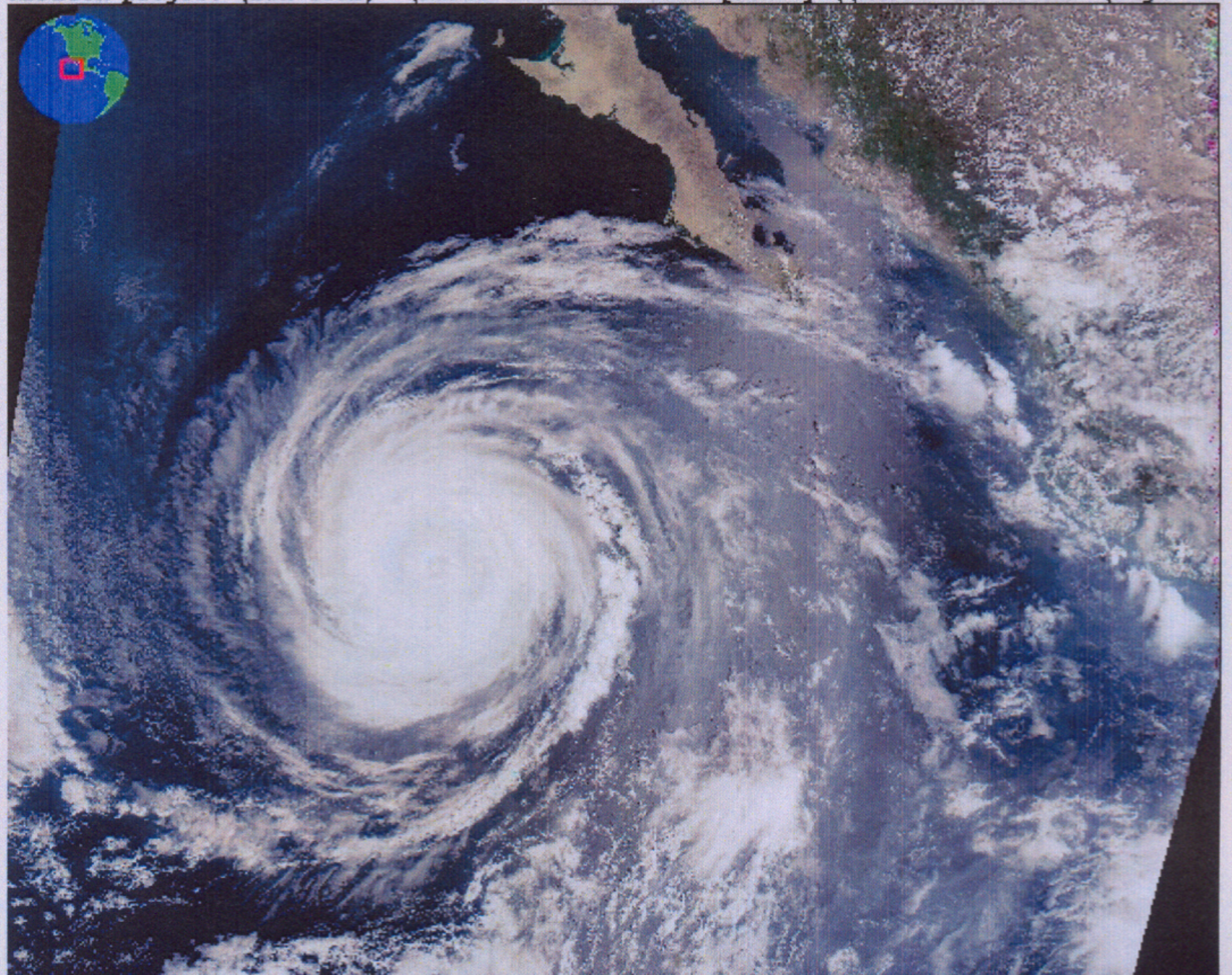


Physical laws governing the flow

- Conservation of mass

  $D\rho/Dt + \rho \, \mathrm{div}\underline{U} = 0$

- Conservation of energy

  $De/Dt - (p/\rho^2) \, D\rho/Dt = Q$

- Conservation of momentum

  $D\underline{U}/Dt + (1/\rho) \, \underline{\mathrm{grad}p} - \underline{g} + 2 \, \underline{\Omega} \wedge \underline{U} = \underline{F}$

- Equation of state

  $f(p, \rho, e) = 0$ $\qquad (p/\rho = rT, \; e = C_v T)$

- Conservation of mass of secondary components (water in the atmosphere, salt in the ocean, chemical species, …)

  $Dq/Dt + q \, \mathrm{div}\underline{U} = S$

These physical laws must be expressed in practice in discretized (and necessarily imperfect) form, both in space and time

Physical laws must in practice be discretized in both space and time

$\Rightarrow$ *numerical models*, which are necessarily imperfect.

Models that are used for large scale weather prediction and for climatological simulation cover the whole volume of the atmosphere. These models are based, at least so far, on the *hydrostatic* hypothesis

in the vertical direction :

$$\partial p/\partial z + \rho g = 0$$

Eliminates momentum equation for vertical direction. In addition, flow is incompressible in coordinates (x, y, p) $\Rightarrow$ number of equations decreased by two units.

Hydrostatic approximation valid, to accuracy $\approx 10^{-4}$, for horizontal scales > 20-30 km

More costly nonhydrostatic models are used for small scale meteorology.

In addition to hydrostatic approximation, the following approximations are (almost) systematically made in global modeling :

- Atmospheric fluid is contained in a spherical shell with negligible thickness. This does not forbid the existence within the shell of a vertical coordinate which, in view of the hydrostatic equation, can be chosen as the pressure p.

- The horizontal component of the Coriolis acceleration due to the vertical motion is neglected (this approximation, sometimes called the *traditional approximation*, is actually a consequence of the previous one).

- Tidal forces are neglected.

These approximations lead to the so-called (and ill-named) *primitive equations*

There exist at present two forms of spatial discretization

- Gridpoint discretization

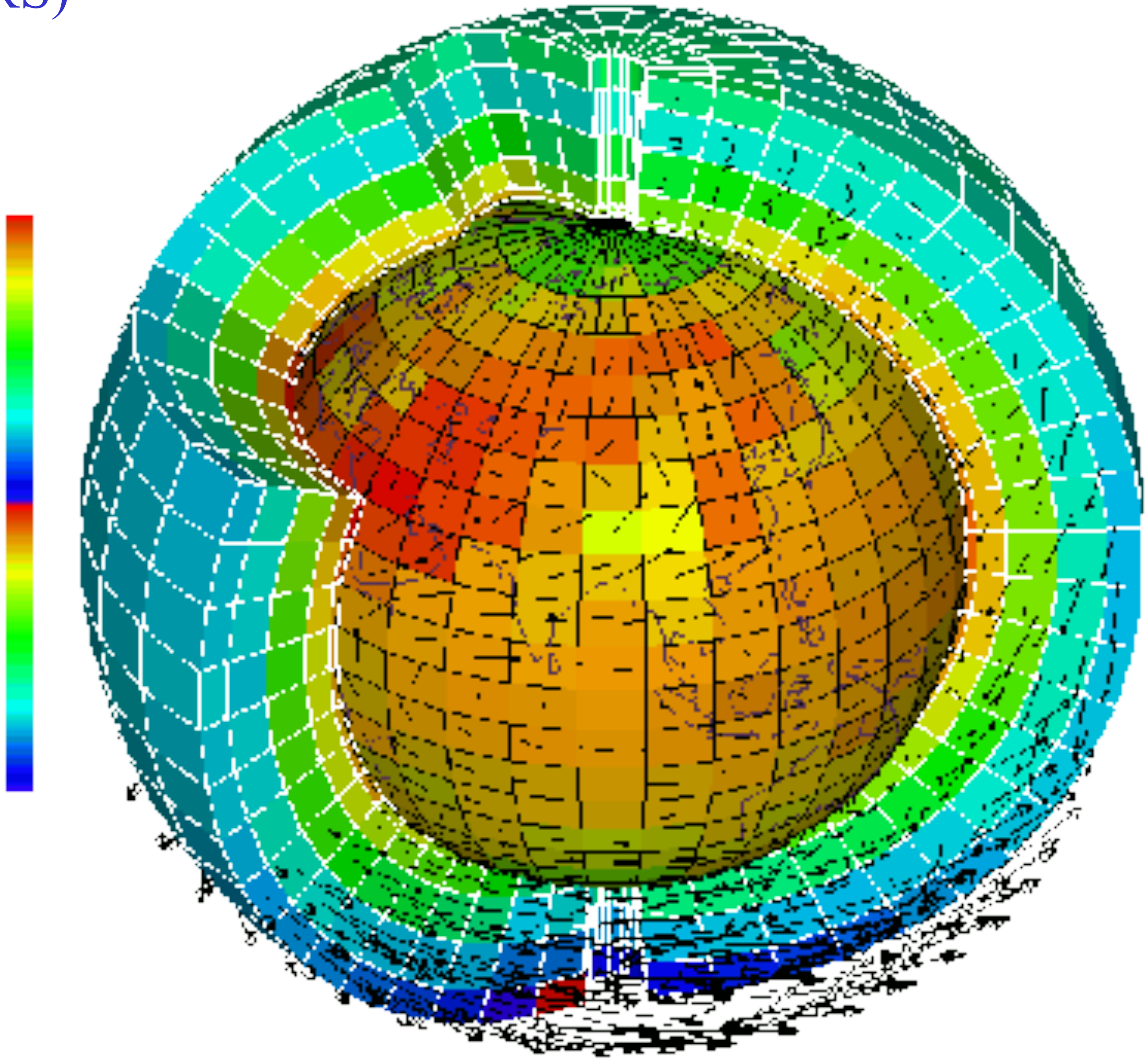
- (Semi-)spectral discretization (mostly for global models, and most often only in the horizontal direction)

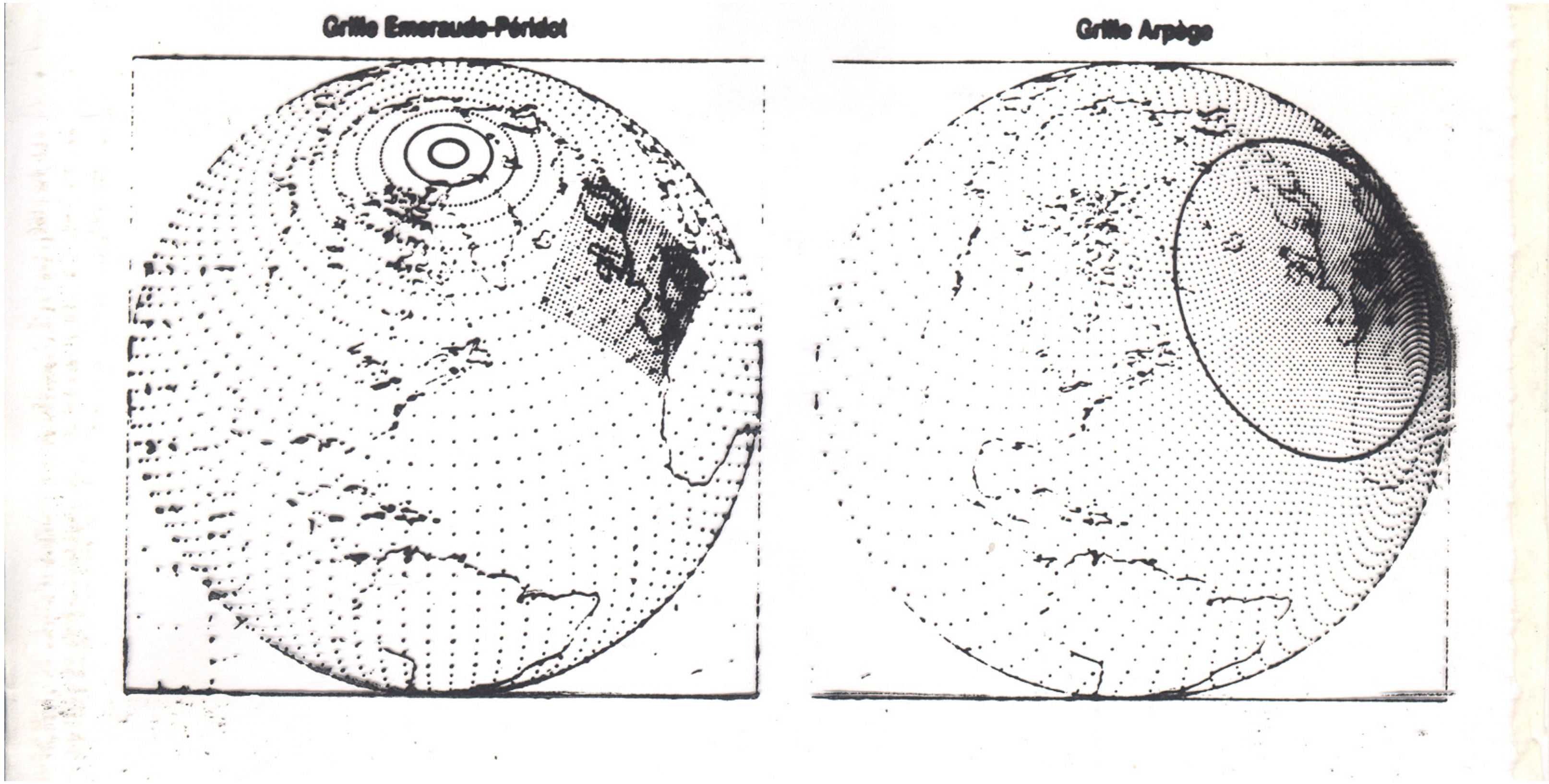
Finite element discretization, which is very common in many forms of numerical modelling, is somtimes used for modelling of the atmosphere, but only in the vertical direction. It is more frequently used for oceanic modelling, where it allows to take into account the complicated geometry of coast-lines.

In gridpoint models, meteorological fields are defined by values at the nodes of a grid covering the physical domain under consideration. Spatial and temporal derivatives are expressed by finite differences.

In spectral models, fields are defined by the coefficients of their expansion along a prescribed set of basic functions. In the case of global meteorological models, those basic functions are the spherical harmonics (eigenfunctions of the laplacian at the surface of the sphere).

A schematic of an Atmospheric General Circulation Model (L. Fairhead /LMD-CNRS)





Grilles de modèles de Météo-France *(La Météorologie)*

Modèles (semi-)spectraux

$$T(\mu\text{=sin(latitude)}, \lambda\text{=longitude}) = \sum_{\substack{0 \le n < \infty \\ -n \le m \le n}} T_n^m Y_n^m(\mu,\lambda)$$

où les $Y_n^m(\mu,\lambda)$ sont les *harmoniques sphériques*

$$Y_n^m(\mu,\lambda) \propto P_n^m(\mu)\exp(im\lambda)$$

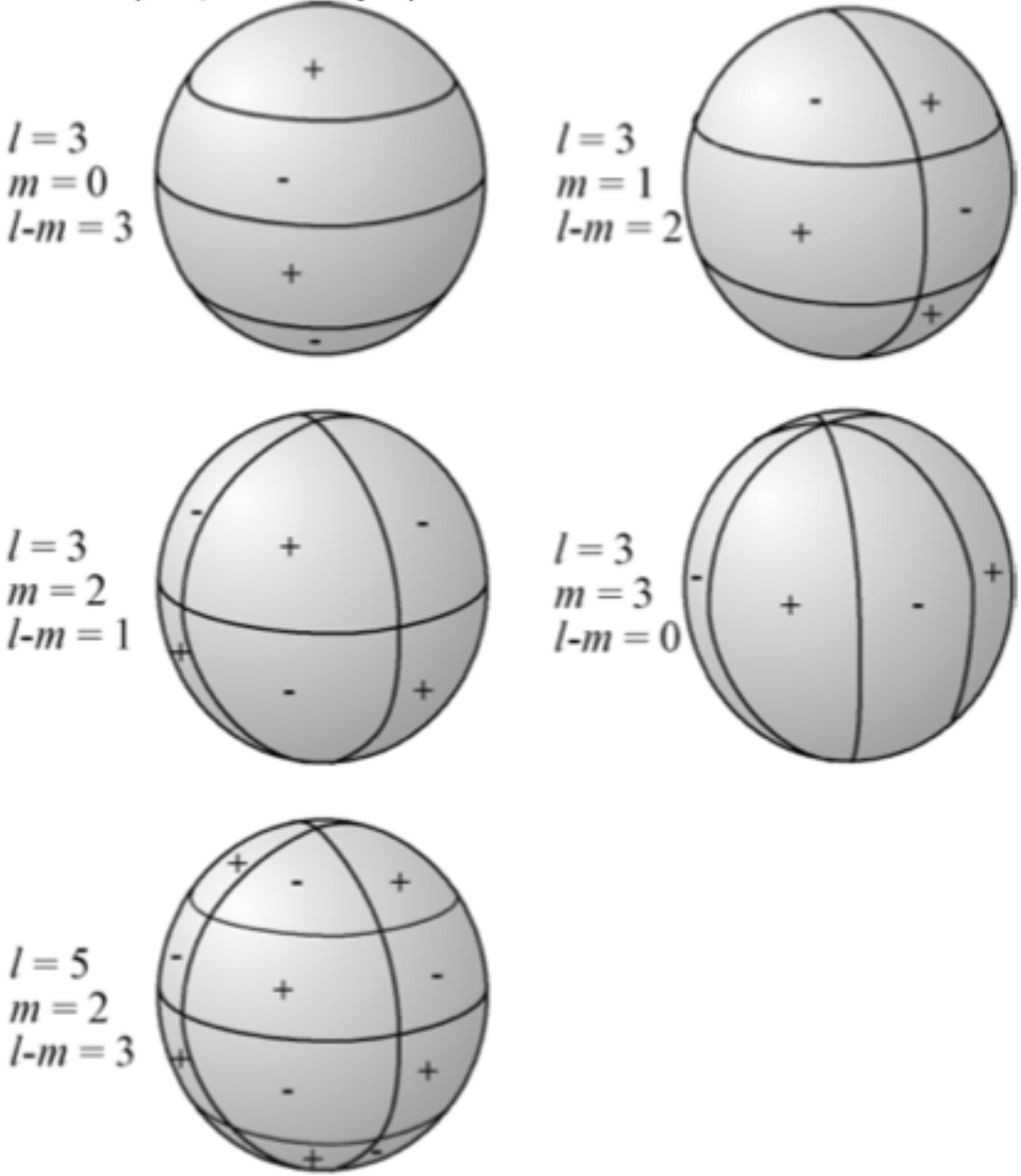
$P_n^m(\mu)$ est la *fonction de Legendre* de deuxième espèce.

$$P_n^m(\mu) \propto (1-\mu^2)^{\frac{m}{2}} \frac{d^{n+m}}{d\mu^{n+m}}(\mu^2-1)^n$$

n et m sont respectivement le *degré* et l'*ordre* de l'harmonique $Y_n^m(\mu,\lambda)$

$n = 0, 1, \ldots \qquad -n \le m \le n$

From Wikipedia, the free encyclopedia



Modèles (semi-)spectraux

Les harmoniques sphériques définissent une base complète orthonormée de l'espace L^2 à la surface S de la sphère.

$$\int_S Y_n^m Y_{n'}^{m'} d\mu d\lambda = \delta_n^{n'} \delta_m^{m'}$$

Relation de Parseval

$$\int_S T^2(\mu,\lambda) d\mu d\lambda = \sum_{\substack{0 \le n < \infty \\ -n \le m \le n}} \left| T_n^m \right|^2$$

Les harmoniques sphériques sont fonctions propres du laplacien à la surface de la sphère

$$\Delta Y_n^m = -n(n+1)Y_n^m$$

Troncature 'triangulaire' *TN* ($n \leq N$, $-n \leq m \leq n$) indépendante du choix d'un axe polaire. Représentation est parfaitement homogène à la surface de la sphère

Calculs non linéaires effectués dans l'espace physique (sur grille latitude-longitude 'gaussienne'). Les transformations requises sont possibles à un coût non prohibitif grâce à l'utilisation de Transformées de Fourier Rapides (*Fast Fourier Transforms*, *FFT*, en anglais). Il existe aussi une version rapide des Transformées de Legendre, relatives à la variable μ.

Hydrostatic approximation allows to take pressure p as independent vertical coordinate

- Flow is incompressible

- Pressure gradient term $(1/\rho)$ grad$_z$ p becomes grad$_p$ $\boldsymbol{\Phi}$, where $\boldsymbol{\Phi} \equiv gz$ is geopotential