École Doctorale des Sciences de l'Environnement d'Île-de-France

Année Universitaire 2018-2019

Modélisation Numérique de l'Écoulement Atmosphérique et Assimilation de Données

Olivier Talagrand

Cours 1

14 Février 2019

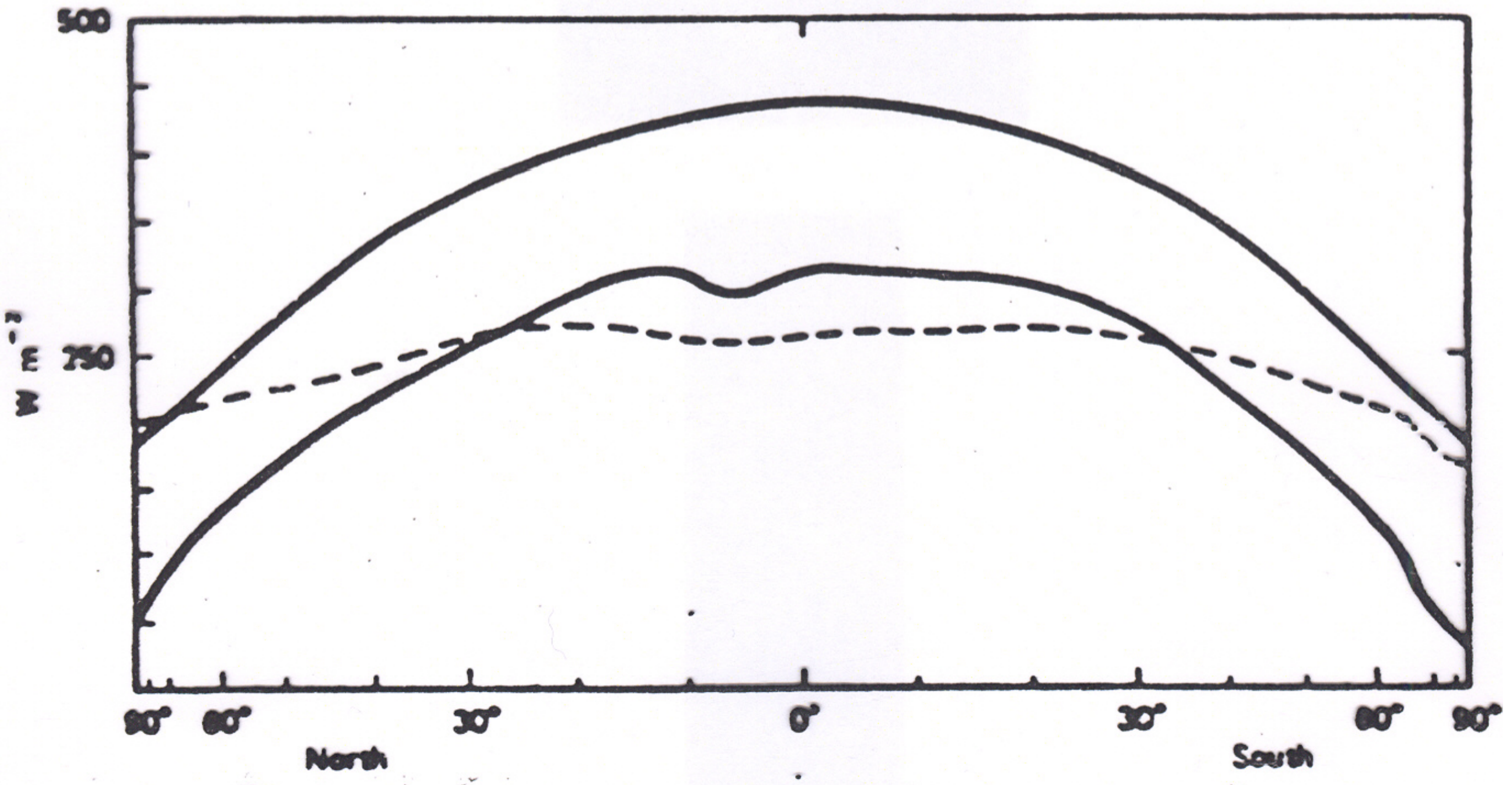
Programme of the course

1. Numerical modeling of the atmospheric flow. The *primitive* equations. Discretization methods. Numerical Weather Prediction. Present performance.
2. The meteorological observation system. The problem of 'assimilation'. Bayesian estimation. Random variables and random functions. Meteorological examples.
3. 'Optimal Interpolation'. Basic properties. Meteorological applications. The theory of *Best Linear Unbiased Estimator*.
4. Advanced assimilation methods.

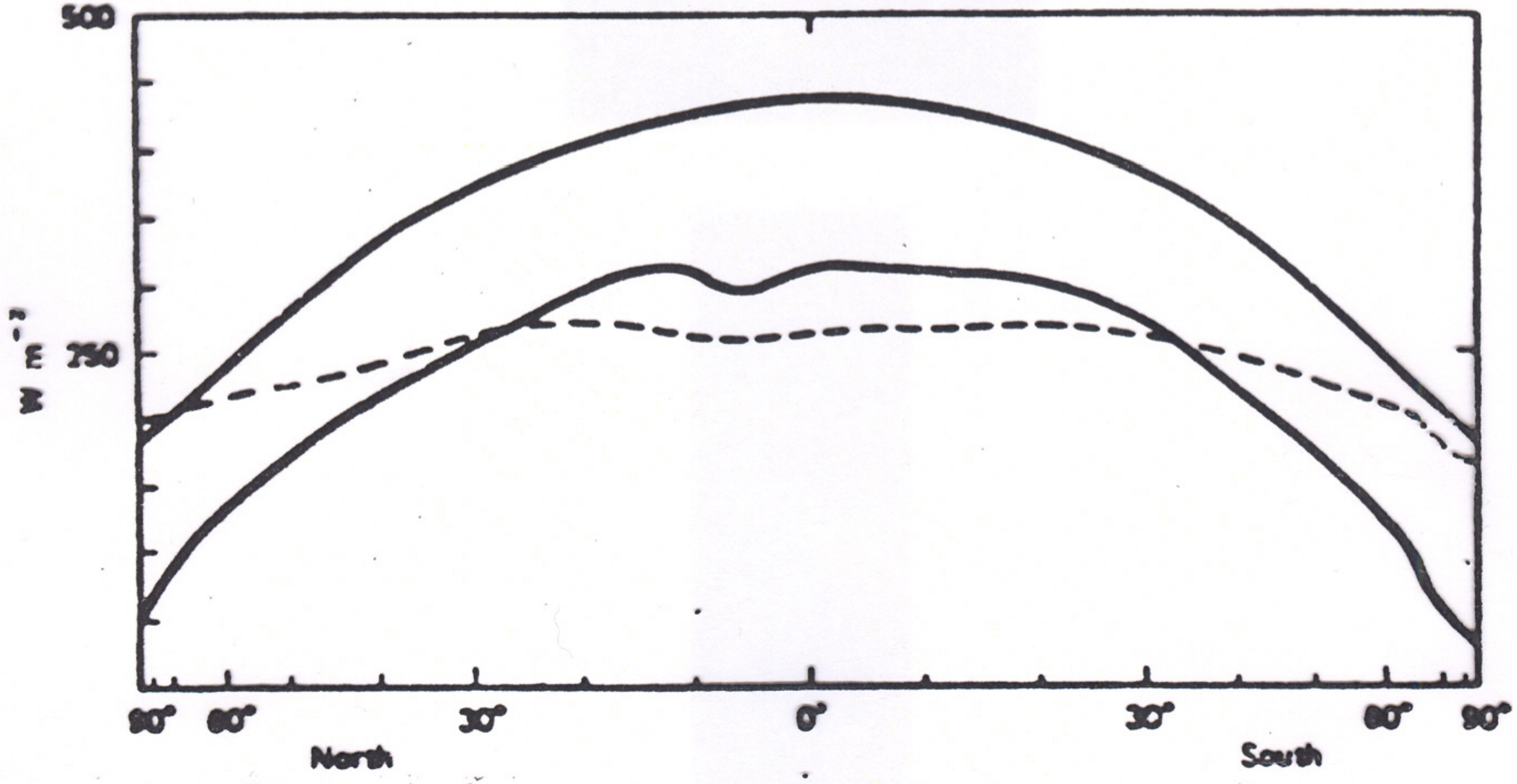
   - Kalman Filter. Ensemble Kalman Filter. Present performance and perspectives.

   - Variational Assimilation. Adjoint Equations. Present performance and perspectives.
5. Advanced assimilation methods (continuation).

   - Bayesian Filters. Theory, present performance and perspectives.



500
250
W m⁻²
90°
60°
30°
0°
30°
60°
90°
North
South



Bilan radiatif de la Terre, moyenné sur un an

Particle moves on sphere with radius R
under the action of a force lying
in meridian plane of the particle

→ Angular momentum wrt axis of rotation conserved.

$$(u + \Omega R \cos\varphi)\, R \cos\varphi = \mathrm{Cst}$$

On Earth, $\Omega \approx 2\pi\, 10^{-5}\, s^{-1}$, $R \approx 6.4\, 10^{6}\, m$.

If $u = 0$ at equator, $u = 329\ ms^{-1}$ at latitude $\varphi = 45°$. If $u = 0$ at $45°$, $u = -232\ ms^{-1}$ at equator.

Hadley, G., 1735, Concerning the cause of the general trade winds, *Philosophical Transactions of the Royal Society*

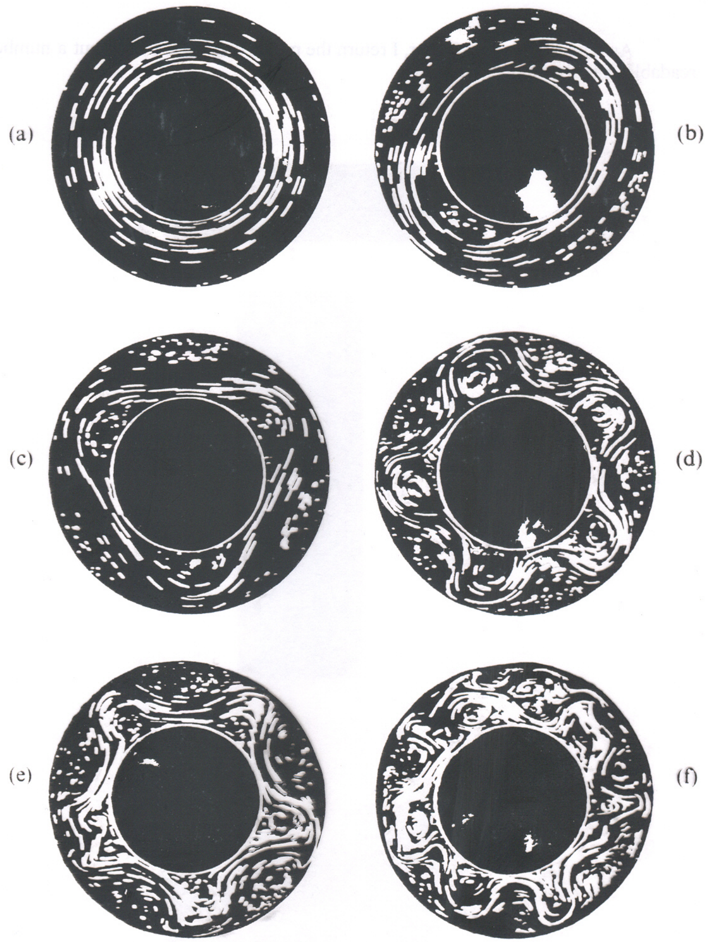


Fig. 10.1. Streak photographs illustrating the dependence of the flow type on rotation rate Ω for a laboratory 'dishpan' experiment. The values of Ω in rad s^{-1} are (a) 0.41; (b) 1.07; (c) 1.21; (d) 3.22; (e) 3.91; (f) 6.4. Working fluid was a water–glycerol solution of mean density 1.037 g cm^{-3} and kinematic viscosity 1.56×10^{-2} cm^2 s^{-1}. The streak photographs show the flow at a depth of 0.5 cm below the free upper surface (see also problem 10.1.) (From Hide & Mason, 1975)

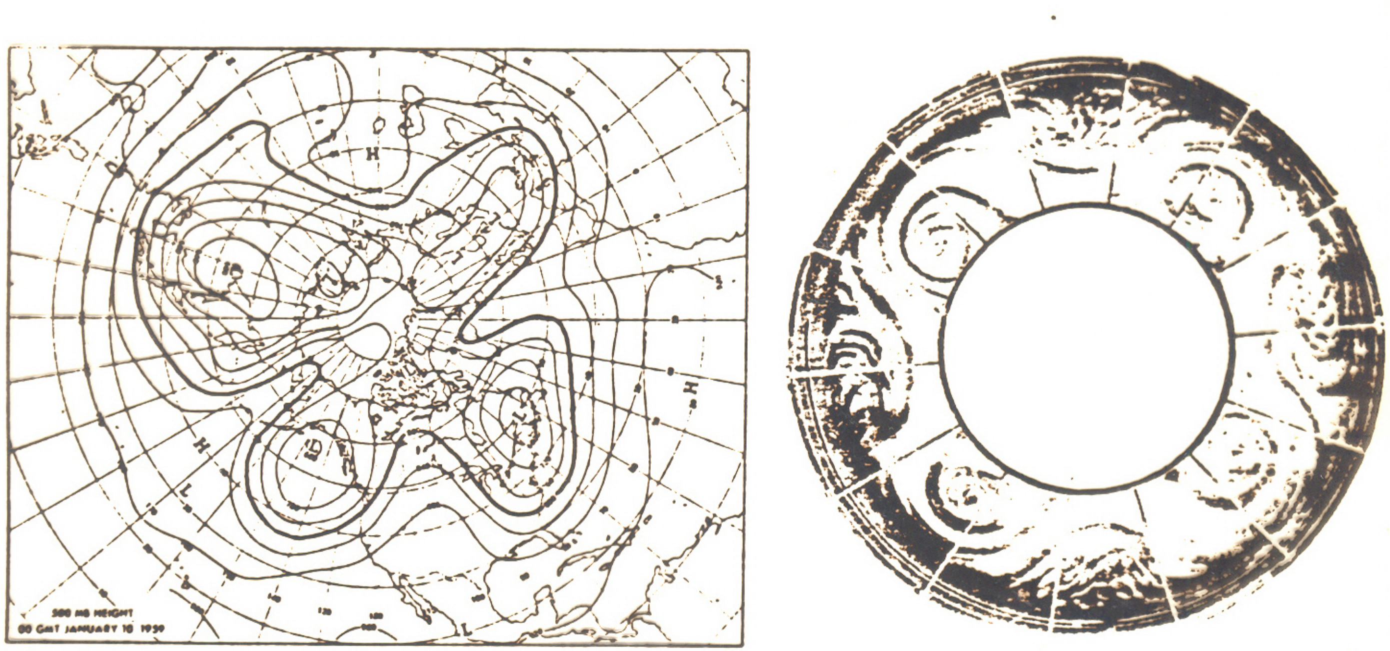
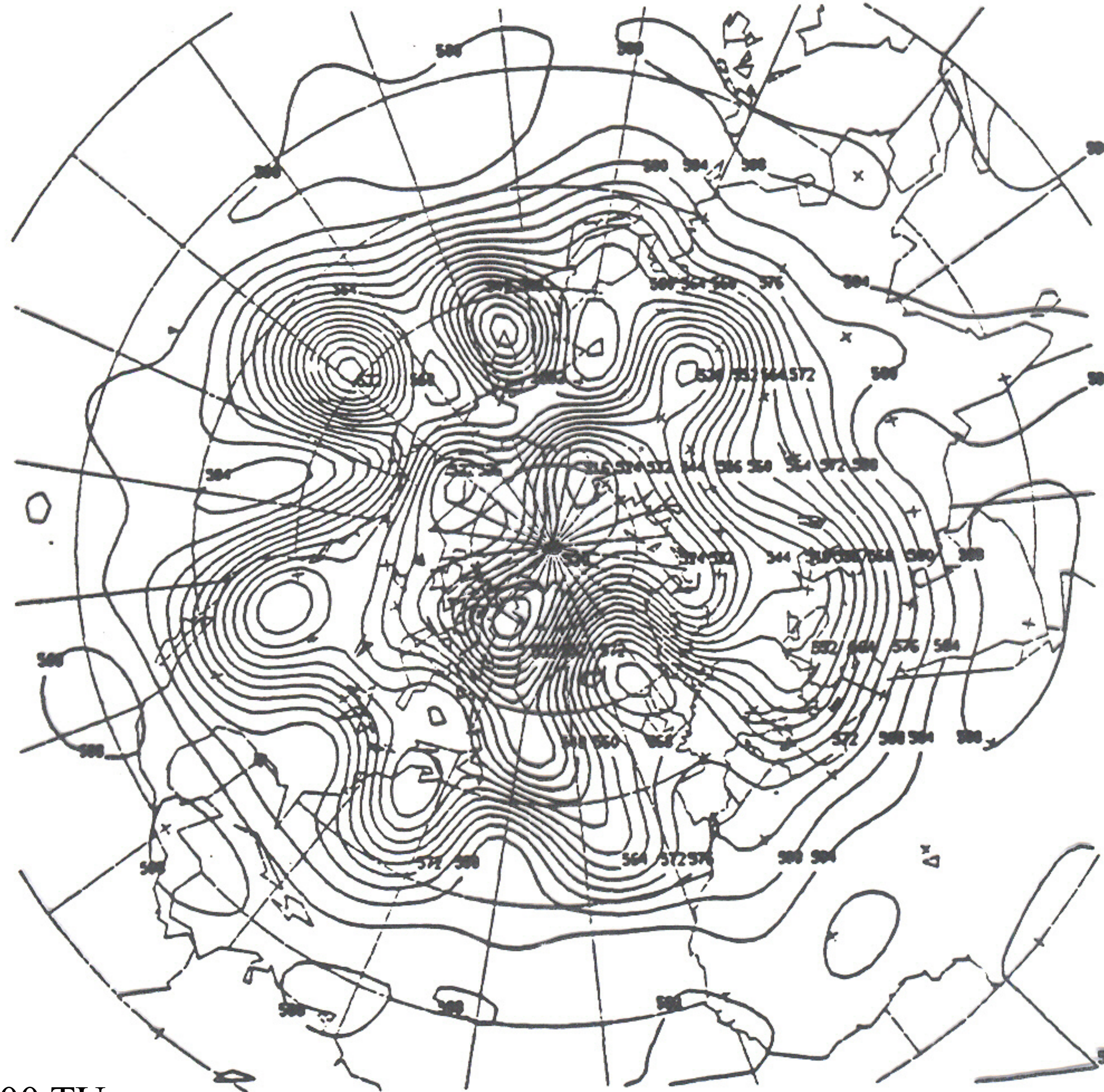
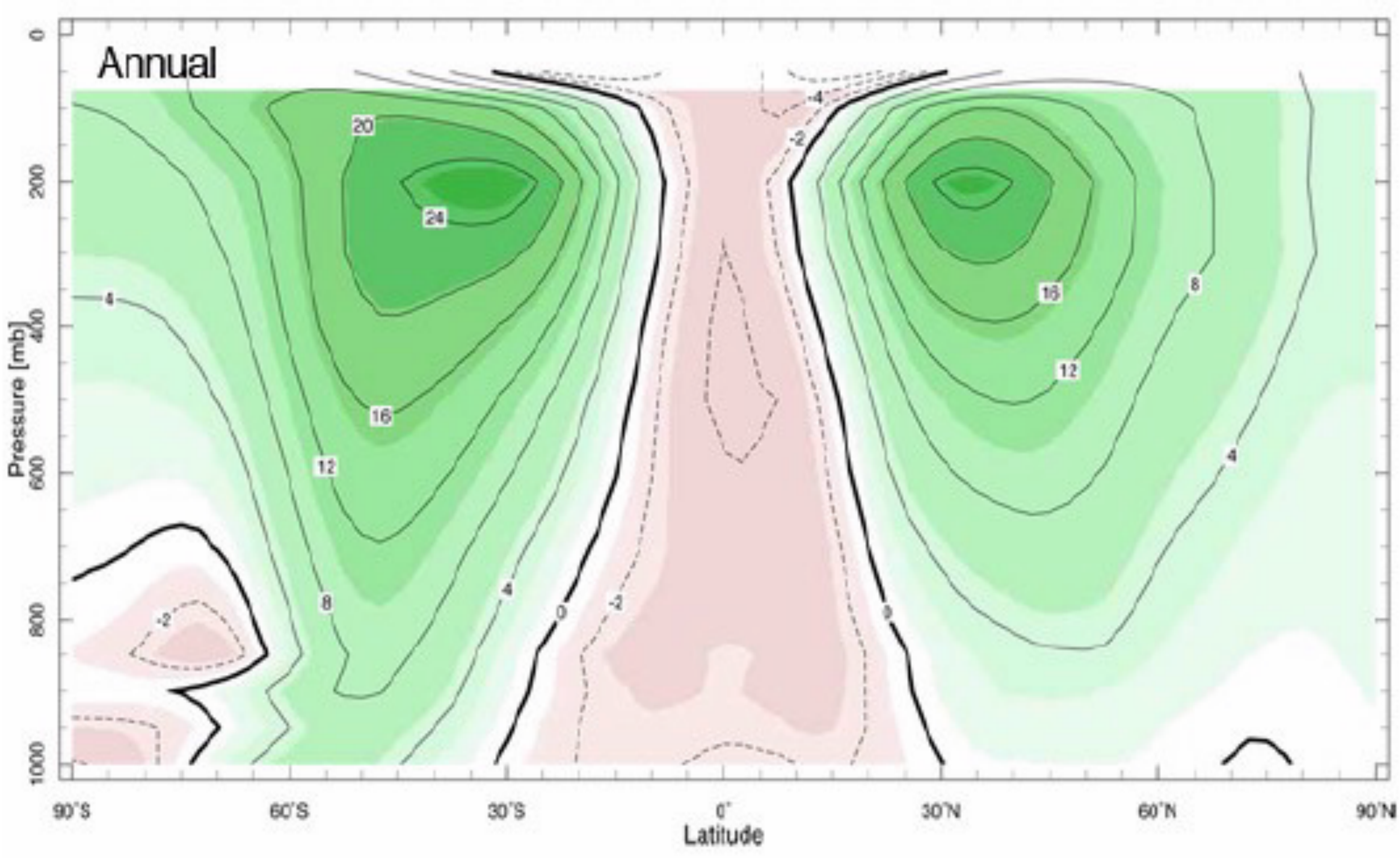


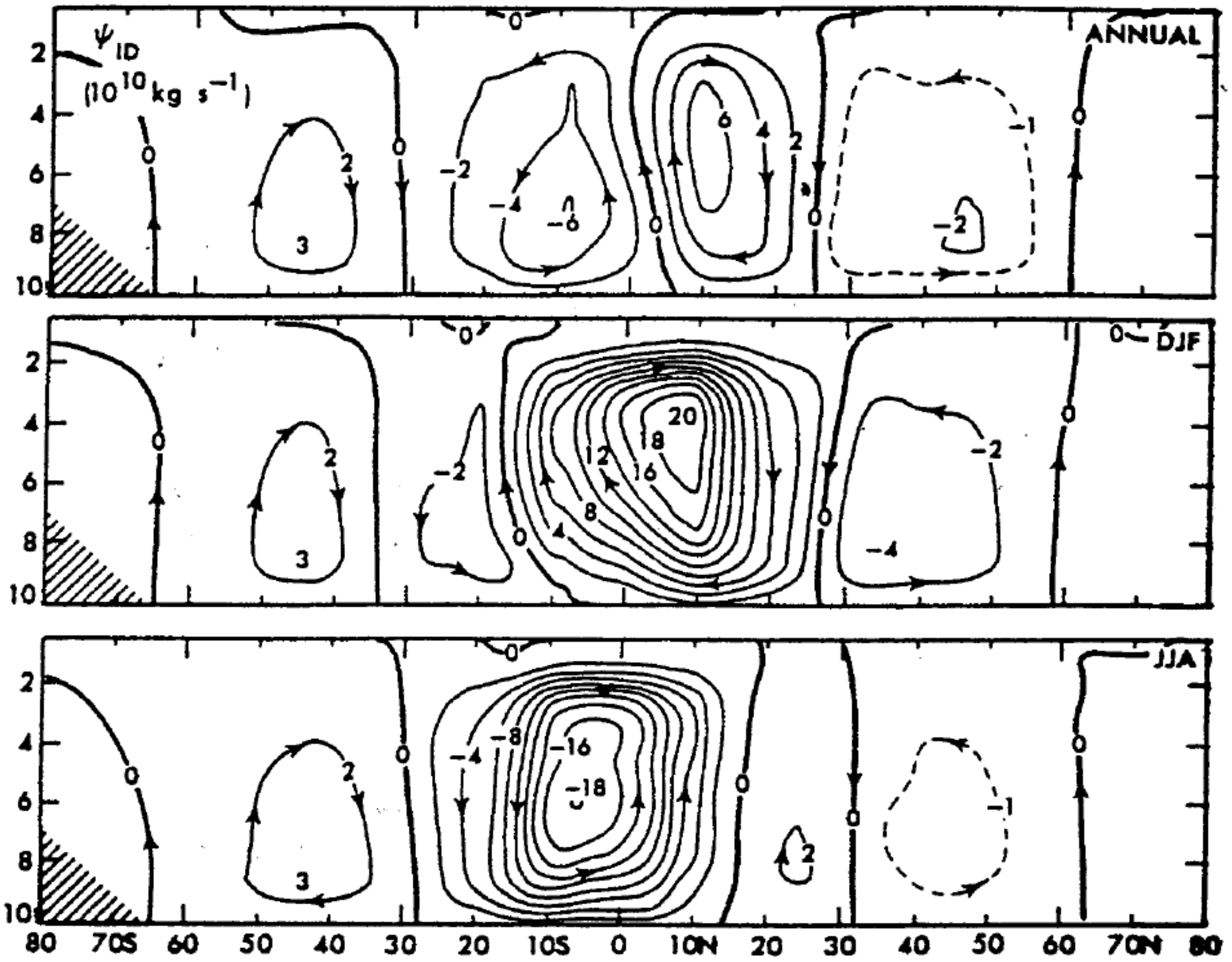
Figure 2. Comparison shows similarities between the global 500 mb pressure pattern in the upper atmosphere of the Northern Hemisphere and a four-wave pattern in the laboratory. (Laboratory flow conditions were similar to those in Fig. 1, except $\Omega = 1.95$ radians per sec.) In the atmosphere the flow is approximately parallel to the isobars (the flow is to the right, from high to low pressure), with speed inversely proportional to the spacing. Changes in the wave pattern have a significant effect on large-scale weather and climate.



26/04/1984, 00/00 TU



Vent zonal; moyenne longitudinale annuelle (m.s^{-1})
http://paoc.mit.edu/labweb/notes/chap5.pdf,
Atmosphere, Ocean and Climate Dynamics, by J. Marshall and R. A. Plumb, International Geophysics, Elsevier)



Peixoto and Oort, 1992, *The Physics of Climate,* Springer-Verlag

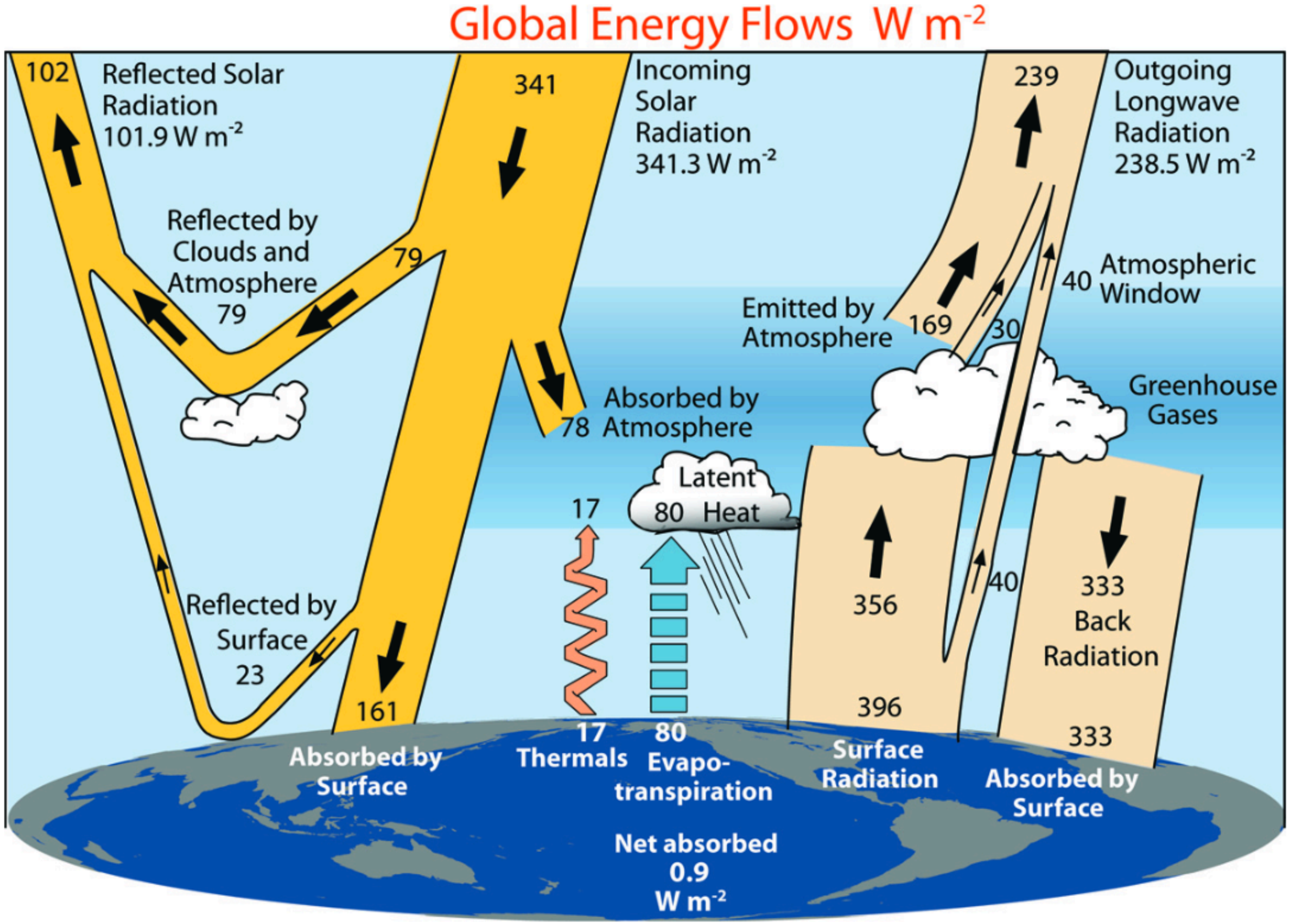
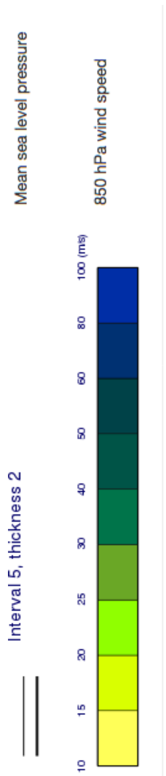
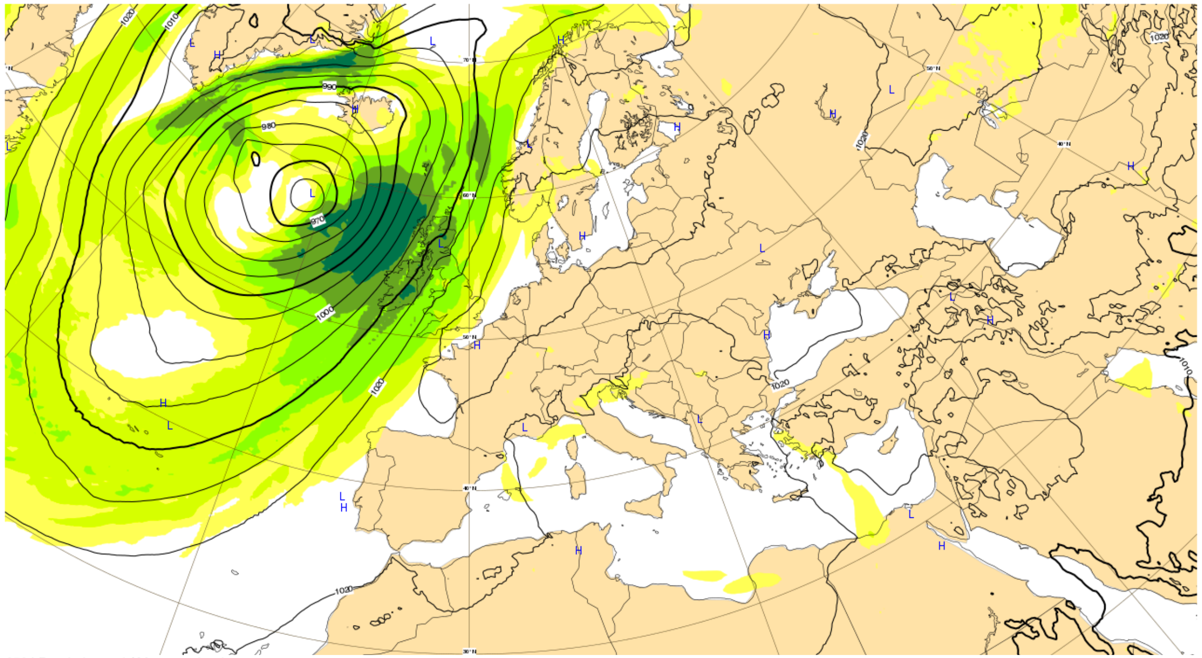


Fig. 1. The global annual mean Earth's energy budget for the Mar 2000 to May 2004 period (W m^{-2}). The broad arrows indicate the schematic flow of energy in proportion to their importance.

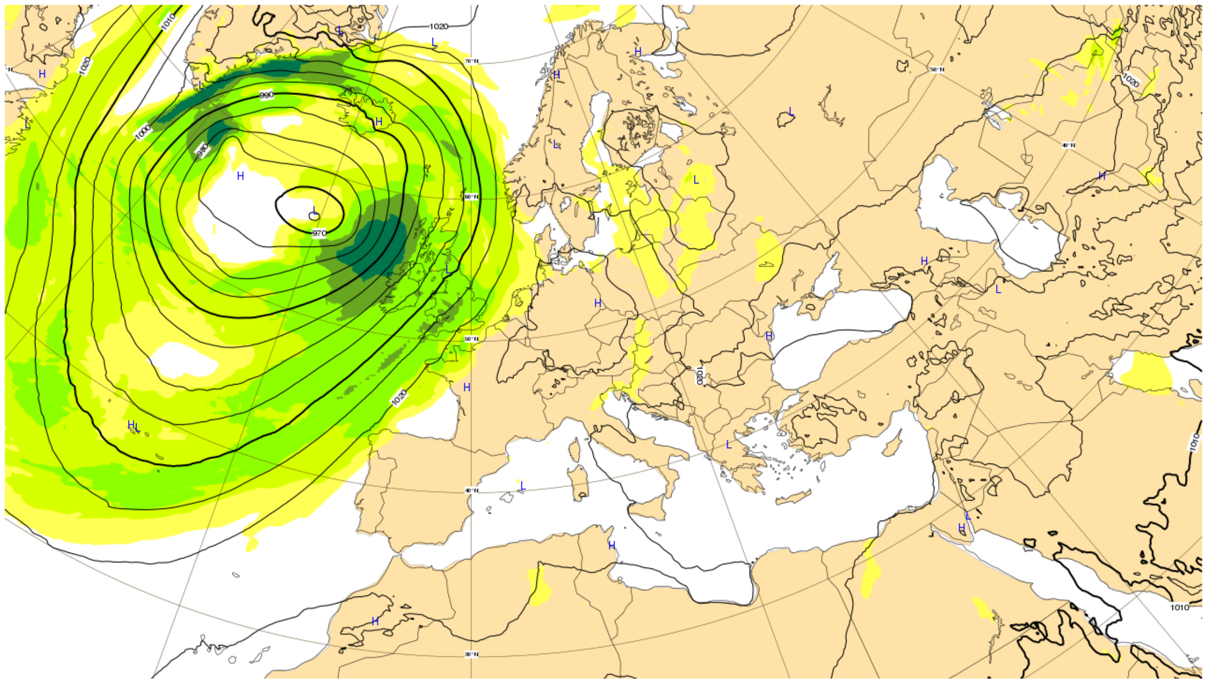
Trenberth *et al.*, *BAMS*, 2009



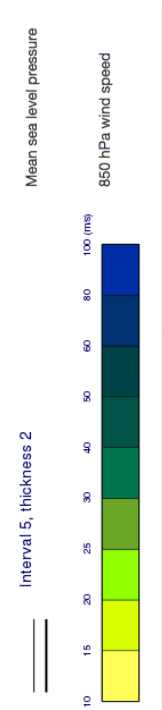
850 hPa wind speed / Mean sea level pressure
Thursday 12 Apr, 12 UTC T+120 Valid: Tuesday 17 Apr, 12 UTC



850 hPa wind speed / Mean sea level pressure
Tuesday 17 Apr, 12 UTC T+0 Valid: Tuesday 17 Apr, 12 UTC



2018



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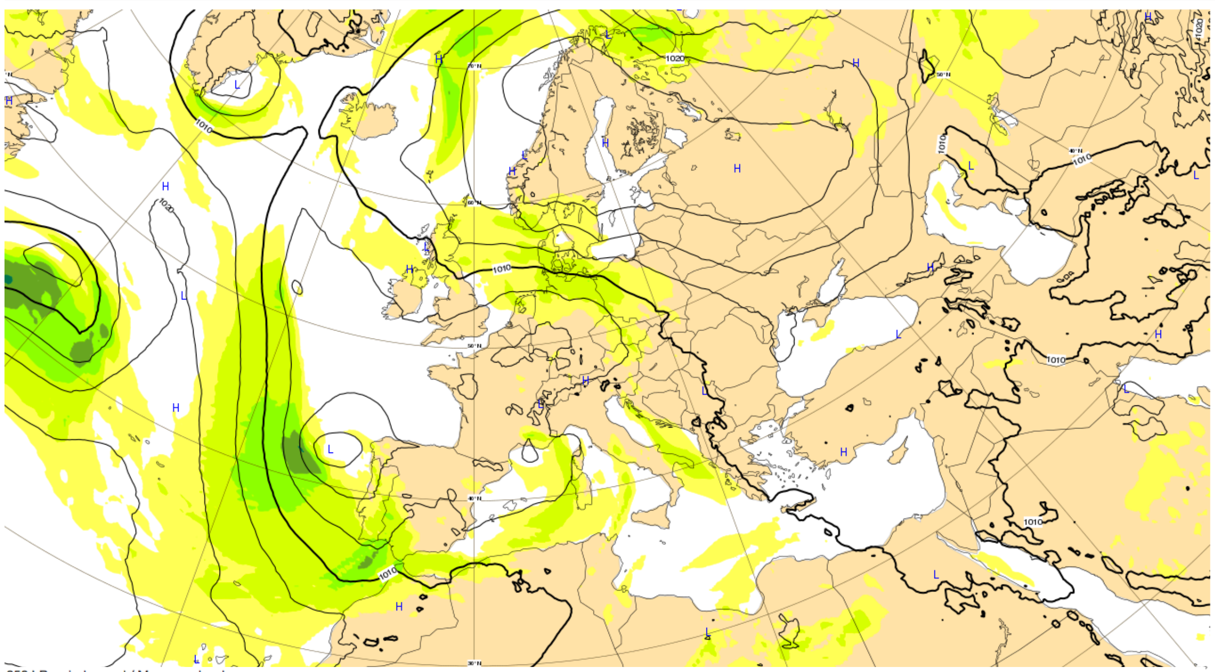

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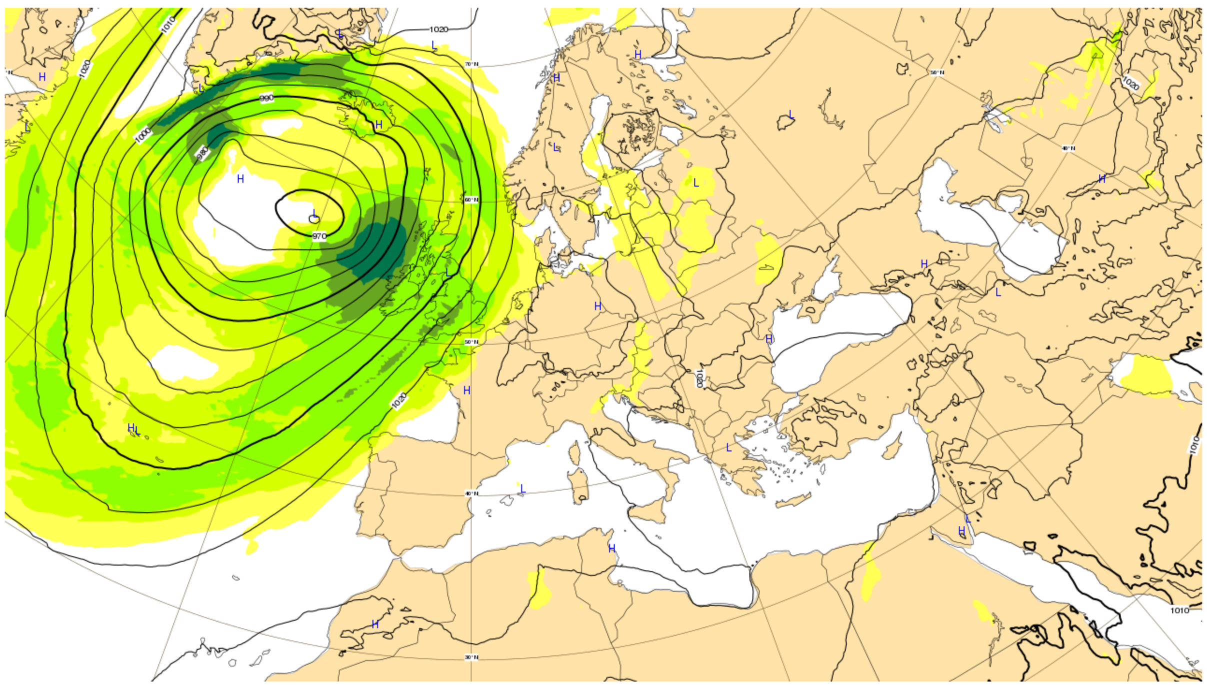


2018

Fig. 1: Members of day 7 forecast of 500 hPa geopotential height for the ensemble originated from 25 January 1993.

Figure 6 Hurricane Katrina mean-sea-level-pressure (MSLP) analysis for 12 UTC of 29 August 2005 and t+84h high-resolution and EPS forecasts started at 00 UTC of 26 August:

1st row: *1^{st} panel: MSLP analysis for 12 UTC of 29 Aug*
2^{nd} panel: MSLP t+84h T_L511L60 forecast started at 00 UTC of 26 Aug
3^{rd} panel: MSLP t+84h EPS-control T_L255L40 forecast started at 00 UTC of 26 Aug
Other rows: 50 EPS-perturbed T_L255L40 forecast started at 00 UTC of 26 Aug.

The contour interval is 5 hPa, with shading patters for MSLP values lower than 990 hPa.

Pourquoi les météorologistes ont-ils tant de peine à prédire le temps avec quelque certitude ? Pourquoi les chutes de pluie, les tempêtes elles-mêmes nous semblent-elles arriver au hasard, de sorte que bien des gens trouvent tout naturel de prier pour avoir la pluie ou le beau temps, alors qu'ils jugeraient ridicule de demander une éclipse par une prière ?[...] un dixième de degré en plus ou en moins en un point quelconque, le cyclone éclate ici et non pas là, et il étend ses ravages sur des contrées qu'il aurait épargnées. Si on avait connu ce dixième de degré, on aurait pu le savoir d'avance, mais les observations n'étaient ni assez serrées, ni assez précises, et c'est pour cela que tout semble dû à l'intervention du hasard.

H. Poincaré, *Science et Méthode*, Paris, 1908

Why have meteorologists such difficulties in predicting the weather with any certainty ? Why is it that showers and even storms seem to come by chance, so that many people think it is quite natural to pray for them, though they would consider it ridiculous to ask for an eclipse by prayer ? [...] *a tenth of a degree more or less at any given point, and the cyclone will burst here and not there, and extend its ravages over districts that it would otherwise have spared. If they had been aware of this tenth of a degree, they could have known it beforehand, but the observations were neither sufficiently comprehensive nor sufficiently precise, and that is the reason why it all seems due to the intervention of chance.*

H. Poincaré, *Science et Méthode*, Paris, 1908
(translated Dover Publ., 1952)

Physical laws governing the flow

- Conservation of mass

  $D\rho/Dt + \rho \, \mathrm{div}\underline{U} = 0$

- Conservation of energy

  $De/Dt - (p/\rho^2) \, D\rho/Dt = Q$

- Conservation of momentum

  $D\underline{U}/Dt + (1/\rho) \, \underline{\mathrm{grad}p} - \underline{g} + 2 \, \underline{\Omega} \wedge \underline{U} = \underline{F}$

- Equation of state

  $f(p, \rho, e) = 0$ $(p/\rho = rT, e = C_v T)$

- Conservation of mass of secondary components (water in the atmosphere, salt in the ocean, chemical species, …)

  $Dq/Dt + q \, \mathrm{div}\underline{U} = S$

These physical laws must be expressed in practice in discretized (and necessarily imperfect) form, both in space and time

Physical laws must in practice be discretized in both space and time

$\Rightarrow$ *numerical models*, which are necessarily imperfect.

Models that are used for large scale weather prediction and for climatological simulation cover the whole volume of the atmosphere. These models are based, at least so far, on the *hydrostatic* hypothesis

in the vertical direction :

$$\partial p/\partial z + \rho g = 0$$

Eliminates momentum equation for vertical direction. In addition, flow is incompressible in coordinates $(x, y, p) \Rightarrow$ number of equations decreased by two units.

Hydrostatic approximation valid, to accuracy $\approx 10^{-4}$, for horizontal scales $> 20\text{-}30$ km

More costly nonhydrostatic models are used for small scale meteorology.

Hydrostatic approximation allows to take pressure p as independent vertical coordinate

- Flow is incompressible

- Pressure gradient term $(1/\rho)\ \mathrm{grad}_z\, p$ becomes $\mathrm{grad}_p\, \boldsymbol{\Phi}$, where $\boldsymbol{\Phi} \equiv gz$ is geopotential

There exist at present two forms of spatial discretization

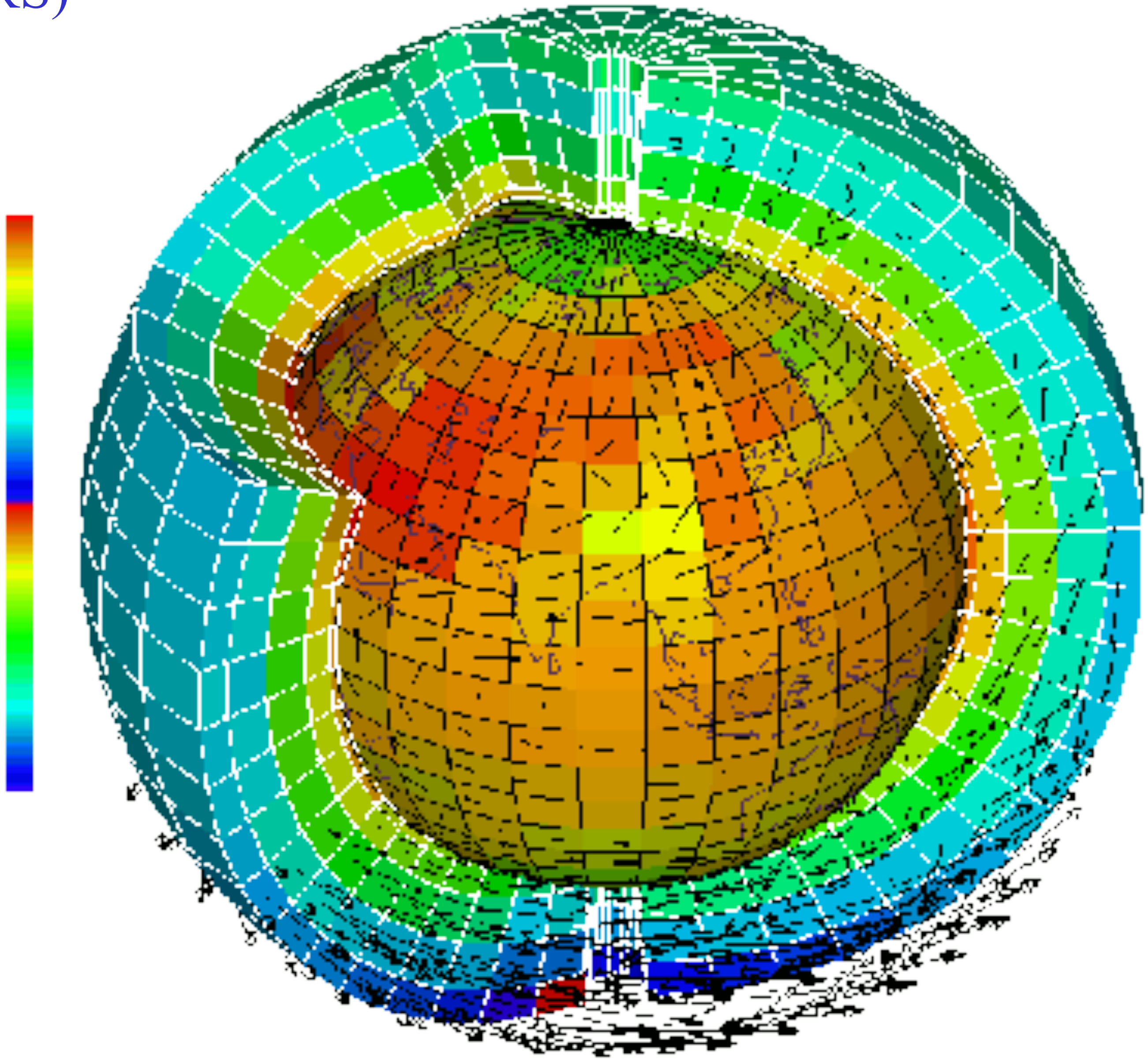
- Gridpoint discretization

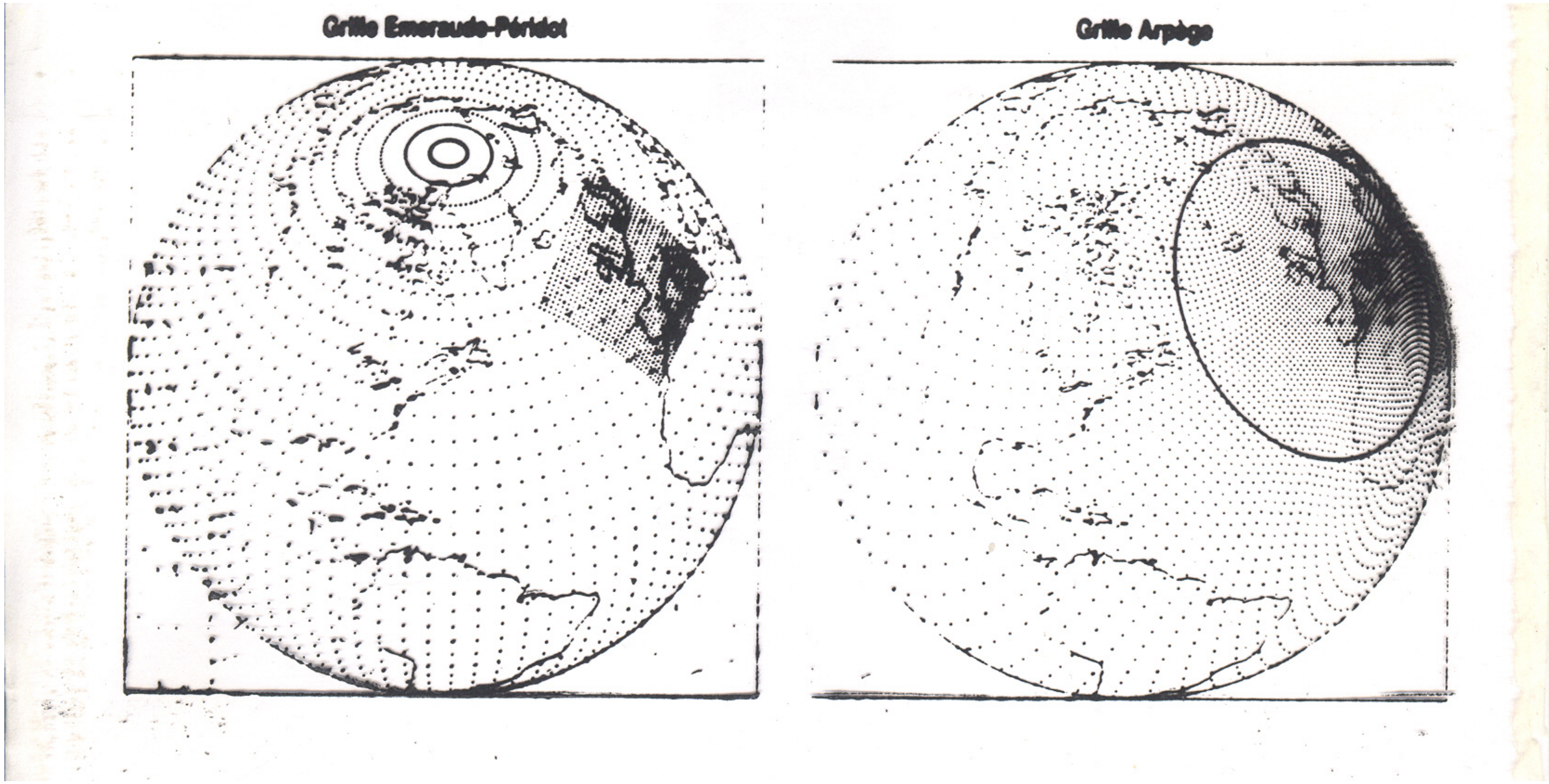
- (Semi-)spectral discretization (mostly for global models, and most often only in the horizontal direction)

Finite element discretization, which is very common in many forms of numerical modelling, is rarely used for modelling of the atmosphere, except for discretization in the vertical direction. It is more frequently used for oceanic modelling, where it allows to take into account the complicated geometry of coast-lines.

In *gridpoint models*, meteorological fields are defined by values at the nodes of a grid covering the physical domain under consideration. Spatial and temporal derivatives are expressed by finite differences.

A schematic of an Atmospheric General Circulation Model (L. Fairhead /LMD-CNRS)





Grilles de modèles de Météo-France *(La Météorologie)*

In *spectral models*, fields are defined by the coefficients of their expansion along a prescribed set of basic functions. In the case of global meteorological models, those basic functions are the spherical harmonics (eigenfunctions of the laplacian at the surface of the sphere).

Modèles (semi-)spectraux

$$T(\mu=\text{sin(latitude)},\ \lambda=\text{longitude}) = \sum_{\substack{0\leq n<\infty \\ -n\leq m\leq n}} T_n^m Y_n^m(\mu,\lambda)$$

où les $Y_n^m(\mu,\lambda)$ sont les *harmoniques sphériques*

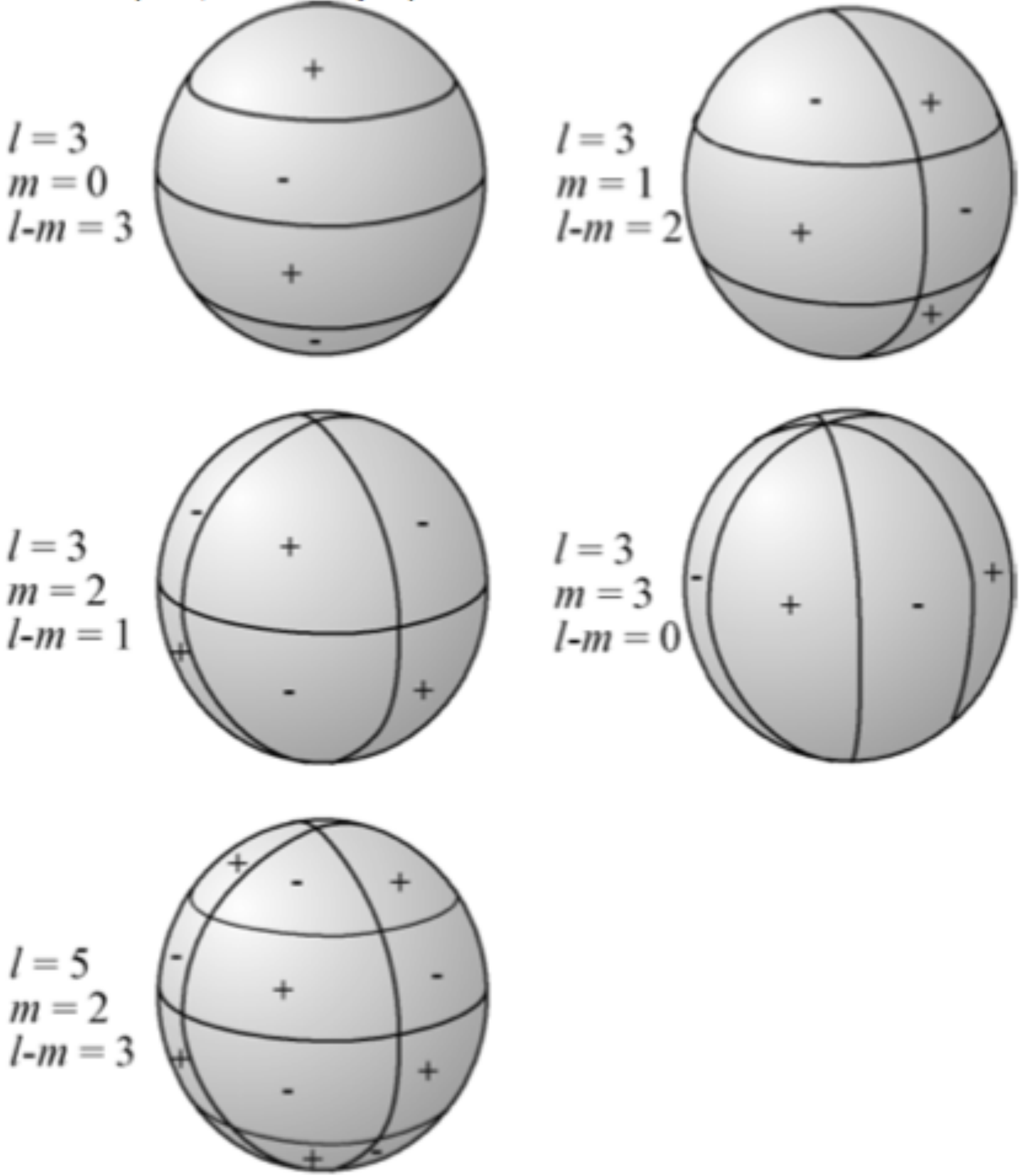
$$Y_n^m(\mu,\lambda) \propto P_n^m(\mu)\exp(im\lambda)$$

$P_n^m(\mu)$ est la *fonction de Legendre* de deuxième espèce

$$P_n^m(\mu) \propto (1-\mu^2)^{\frac{m}{2}} \frac{d^{n+m}}{d\mu^{n+m}}(\mu^2-1)^n$$

n et m sont respectivement le *degré* et l'*ordre* de l'harmonique $Y_n^m(\mu,\lambda)$

$n = 0, 1, \ldots \qquad -n \leq m \leq n$


l = 3
m = 0
l-m = 3
l = 3
m = 1
l-m = 2
l = 3
m = 2
l-m = 1
l = 3
m = 3
l-m = 0
l = 5
m = 2
l-m = 3

Modèles (semi-)spectraux

Les harmoniques sphériques définissent une base complète orthonormée de l'espace L^2 à la surface S de la sphère.

$$\int_S Y_n^m [Y_{n'}^{m'}]^* d\mu d\lambda = \delta_n^{n'} \delta_m^{m'}$$

Relation de Parseval

$$\int_S T^2(\mu,\lambda) d\mu d\lambda = \sum_{\substack{0 \le n < \infty \\ -n \le m \le n}} \left| T_n^m \right|^2$$

Les harmoniques sphériques sont fonctions propres du laplacien à la surface de la sphère

$$\Delta Y_n^m = -n(n+1)Y_n^m$$

Troncature ‘triangulaire’ *TN* ($n \leq N$, $-n \leq m \leq n$) indépendante du choix d’un axe polaire. Représentation est parfaitement homogène à la surface de la sphère

Calculs non linéaires effectués dans l’espace physique (sur grille appropriée, souvent latitude-longitude ‘gaussienne’). Les transformations requises sont possibles à un coût non prohibitif grâce à l’utilisation de Transformées de Fourier Rapides (*Fast Fourier Transforms*, *FFT*, en anglais). Il existe aussi une version rapide des Transformées de Legendre, relatives à la variable μ.

In addition to hydrostatic approximation, the following approximations are (almost) systematically made in global modeling :

- Atmospheric fluid is contained in a spherical shell with negligible thickness. This does not forbid the existence within the shell of a vertical coordinate which, in view of the hydrostatic equation, can be chosen as the pressure p.

- The horizontal component of the Coriolis acceleration due to the vertical motion is neglected (this approximation, sometimes called the *traditional approximation*, is actually a consequence of the previous one).

- Tidal forces are neglected.

These approximations lead to the so-called (and ill-named) *primitive equations*

Pressure p, although convenient for writing down the equations, is in fact rather inconvenient because lower boundary is not fixed in (x, y, p)-space.

So-called σ-coordinate. $\sigma \equiv p/p_S$, where p_S is pressure at ground level.

'Hybrid' coordinate.

Cours à venir

~~Jeudi 14 Février~~

Jeudi 21 Février (**)

Jeudi 28 Février

Jeudi 7 Mars

Vendredi 15 Mars

Jeudi 21 Mars (*)

Jeudi 28 Mars (*)

De 10h00 à 12h30, Département de Géosciences, École Normale Supérieure, 24, rue Lhomond, Paris 5, Salle de la Serre, 5ième étage,

(*) Salle E314, 3ième étage

(**)) Salle E350, 3ième étage

Particle moves on sphere with radius R
under the action of a force lying
in meridian plane of the particle

→ Angular momentum wrt axis of rotation conserved.

$$(u + \Omega R \cos\varphi) R \cos\varphi = \text{Cst}$$

On Earth, $\Omega \approx 2\pi\, 10^{-5}\, s^{-1}$, $R \approx 6.4\, 10^{6}\, m$.

If $u = 0$ at equator, $u = 329\, ms^{-1}$ at latitude $\varphi = 45°$. If $u = 0$ at $45°$, $u = -232\, ms^{-1}$ at equator.

Hadley, G., 1735, Concerning the cause of the general trade winds, *Philosophical Transactions of the Royal Society*