École Doctorale des Sciences de l'Environnement d'Île-de-France

Année Universitaire 2020-2021

Modélisation Numérique de l'Écoulement Atmosphérique et Assimilation de Données

Olivier Talagrand
Cours 1

26 Mars 2021

Programme of the course

1. Numerical modelling of the atmospheric flow. The *primitive* equations. Discretization methods. Numerical Weather Prediction. Present performance.
2. The meteorological observation system. The problem of 'assimilation'. Bayesian estimation. Random variables and random functions. Meteorological examples.
3. 'Optimal Interpolation'. Basic properties. Meteorological applications. The theory of *Best Linear Unbiased Estimator*.
4. Advanced assimilation methods.

    - Kalman Filter. Ensemble Kalman Filter. Present performance and perspectives.

    - Variational Assimilation. Adjoint Equations. Present performance and perspectives.
5. Advanced assimilation methods (continuation).

    - Bayesian Filters. Theory, present performance and perspectives.
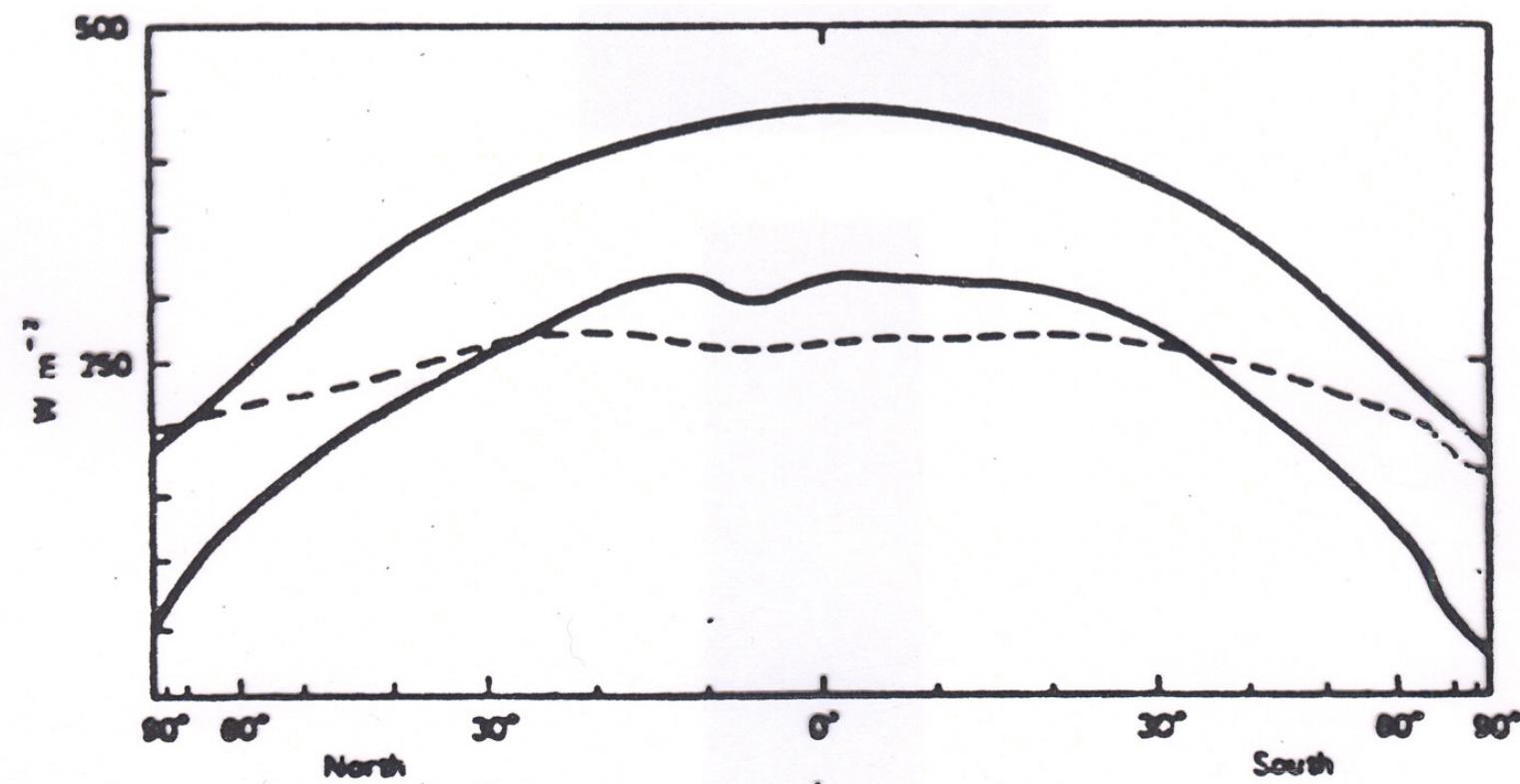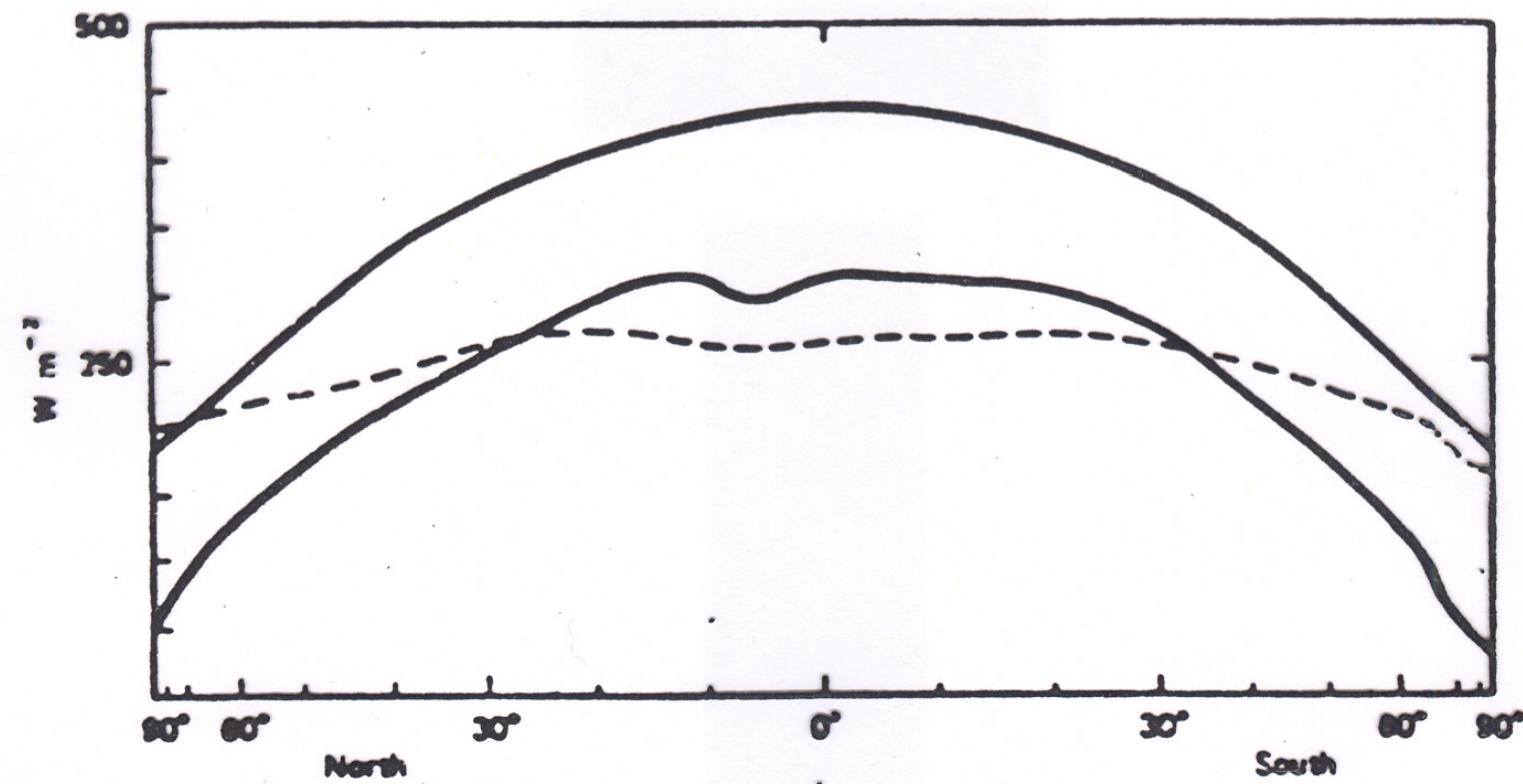
Bilan radiatif de la Terre, moyenné sur un an
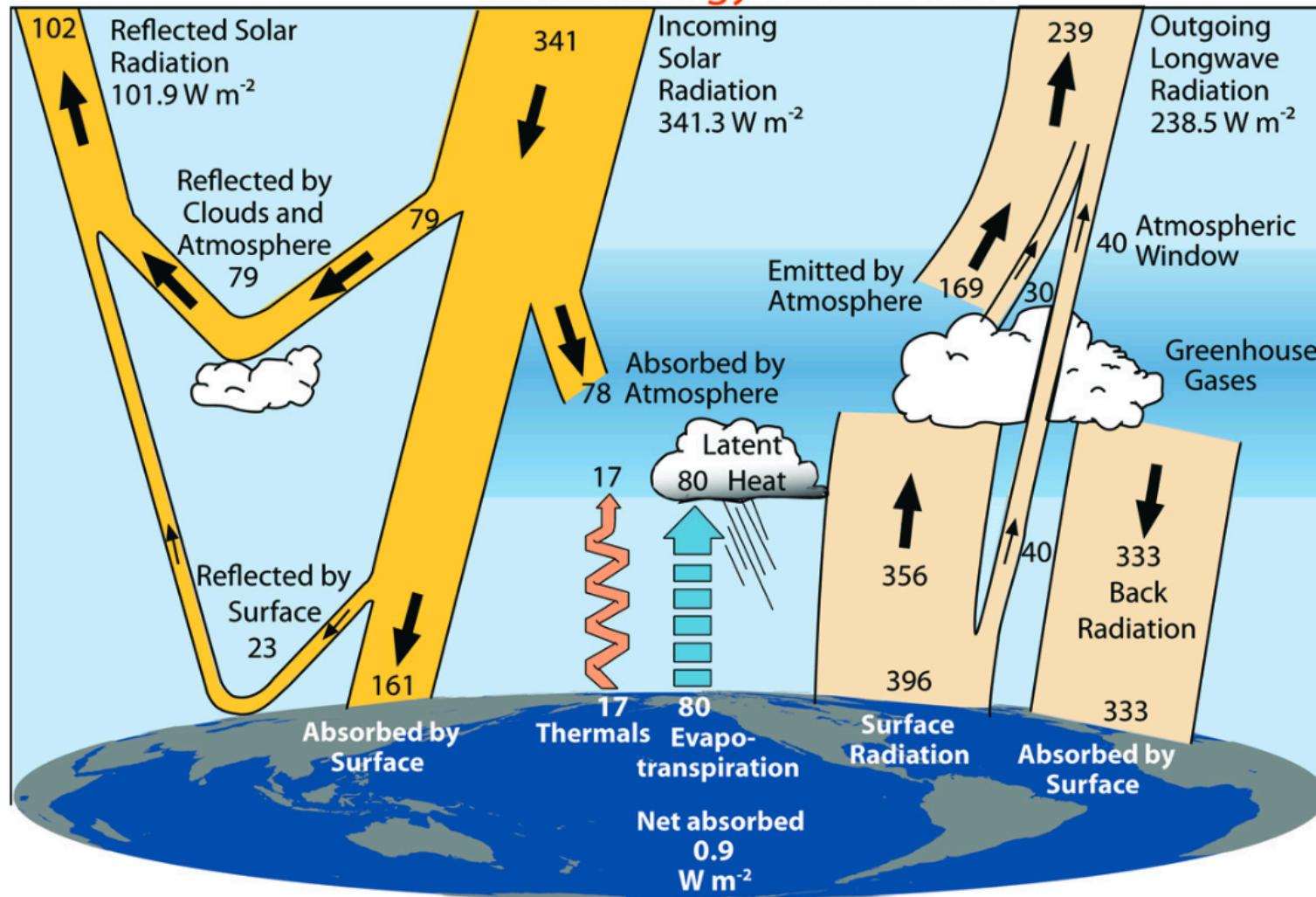
FIG. 1. The global annual mean Earth's energy budget for the Mar 2000 to May 2004 period (W m⁻²). The broad arrows indicate the schematic flow of energy in proportion to their importance.

Trenberth *et al.*, *BAMS*, 2009
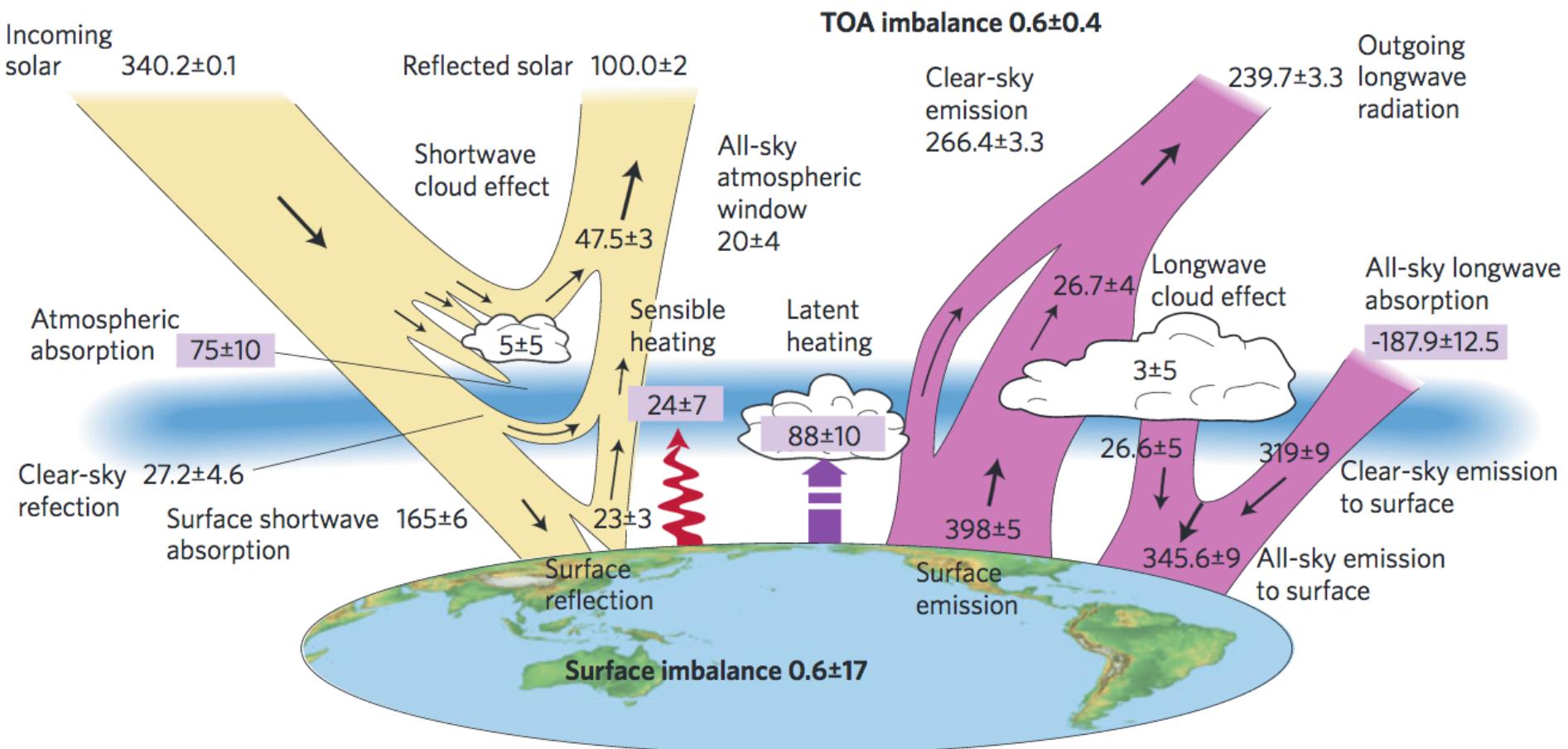
Annual mean energy budget of atmosphere for period 2000–2010. Unit W.m^{-2}

Stephens *et al.*, *Nature Geoscience,* 2012
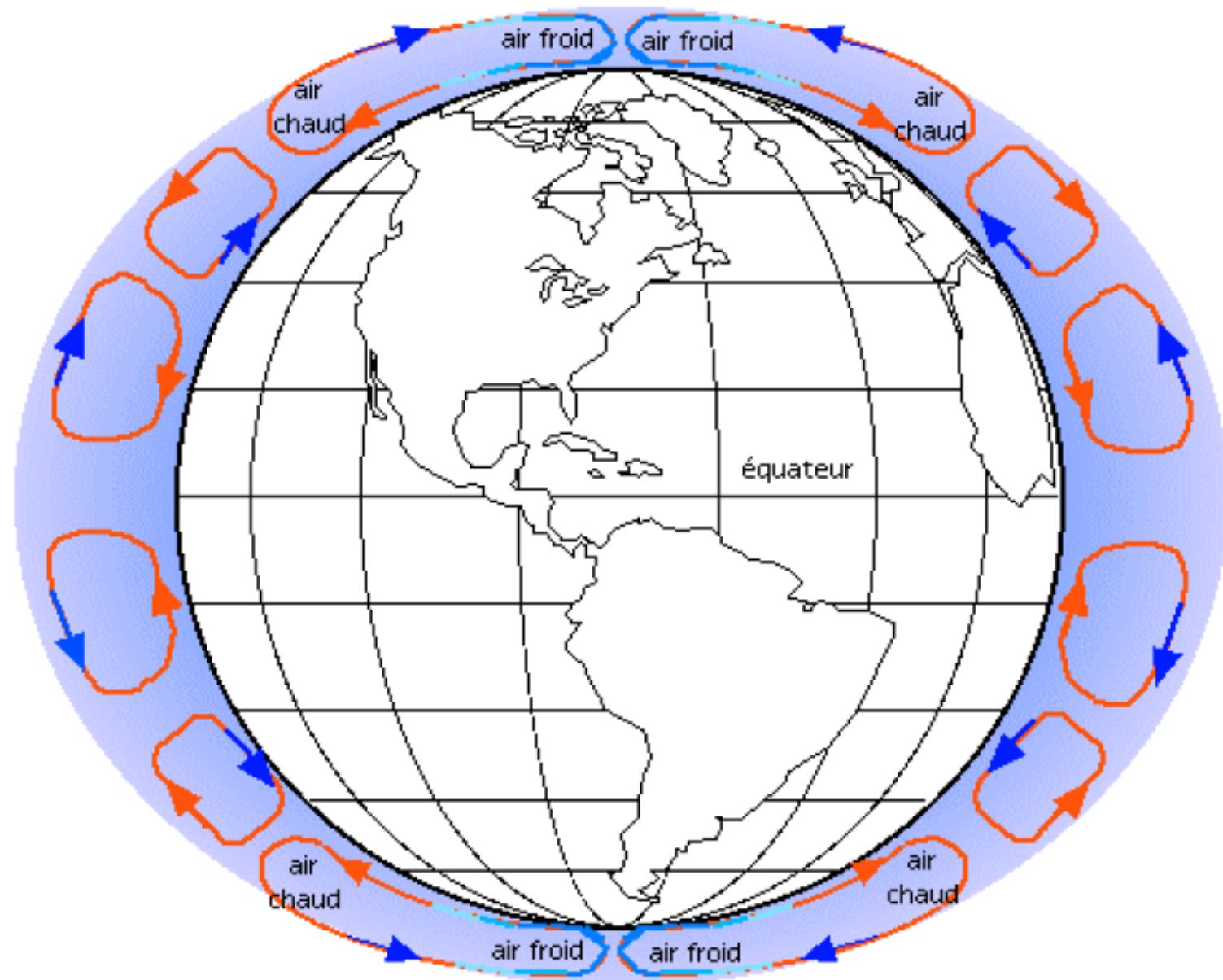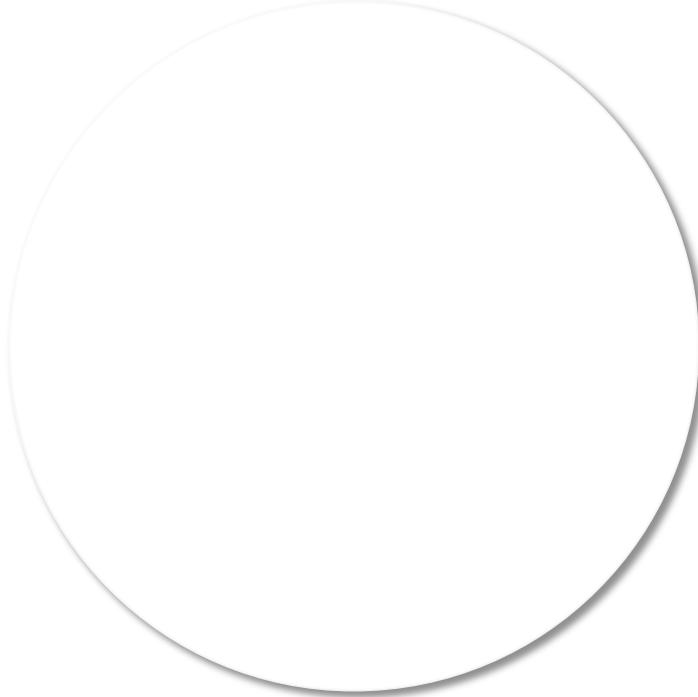
Particle moves on sphere with radius R
under the action of a force lying
in meridian plane of the particle

$\rightarrow$ Angular momentum wrt axis of rotation conserved.

$$(u + \Omega R \cos\varphi)\, R \cos\varphi = \text{Cst}$$

On Earth, $\Omega \approx 2\pi\, 10^{-5}\ s^{-1}$, $R \approx 6.4\ 10^{6}\ m$.

If $u = 0$ at equator, $u = 329\ ms^{-1}$ at latitude $\varphi = 45°$. If $u = 0$ at $45°$, $u = -232\ ms^{-1}$
at equator.

Hadley, G., 1735, Concerning the cause of the general trade winds, *Philosophical Transactions of the Royal Society*
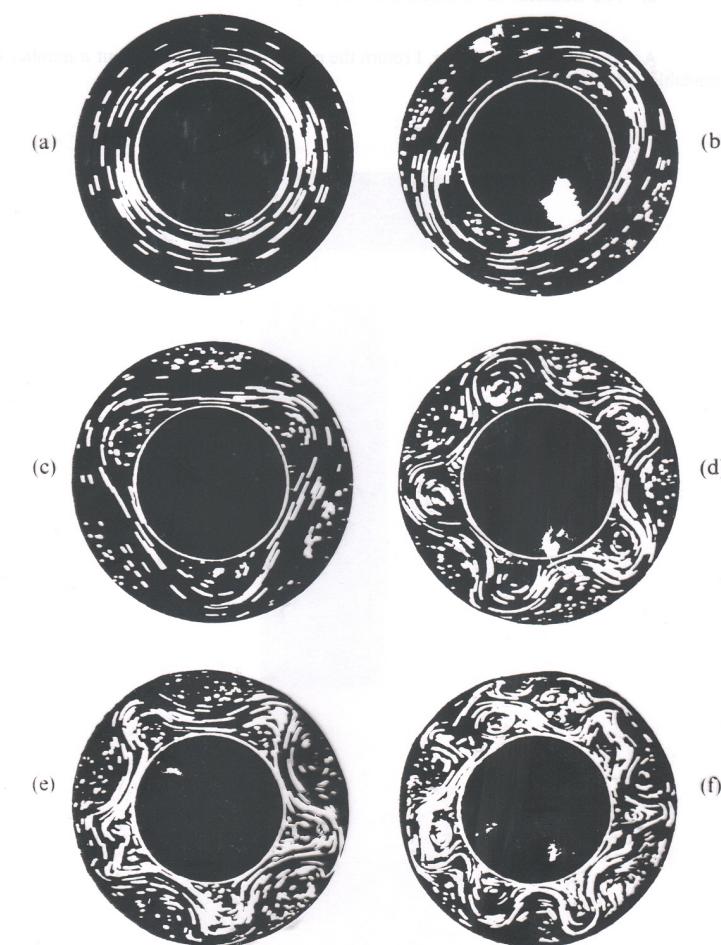
Fig. 10.1. Streak photographs illustrating the dependence of the flow type on rotation rate Ω for a laboratory 'dishpan' experiment. The values of Ω in rad s^{-1} are (a) 0.41; (b) 1.07; (c) 1.21; (d) 3.22; (e) 3.91; (f) 6.4. Working fluid was a water–glycerol solution of mean density 1.037 g cm^{-3} and kinematic viscosity 1.56×10^{-2} cm^2 s^{-1}. The streak photographs show the flow at a depth of 0.5 cm below the free upper surface (see also problem 10.1.) (From Hide & Mason, 1975)
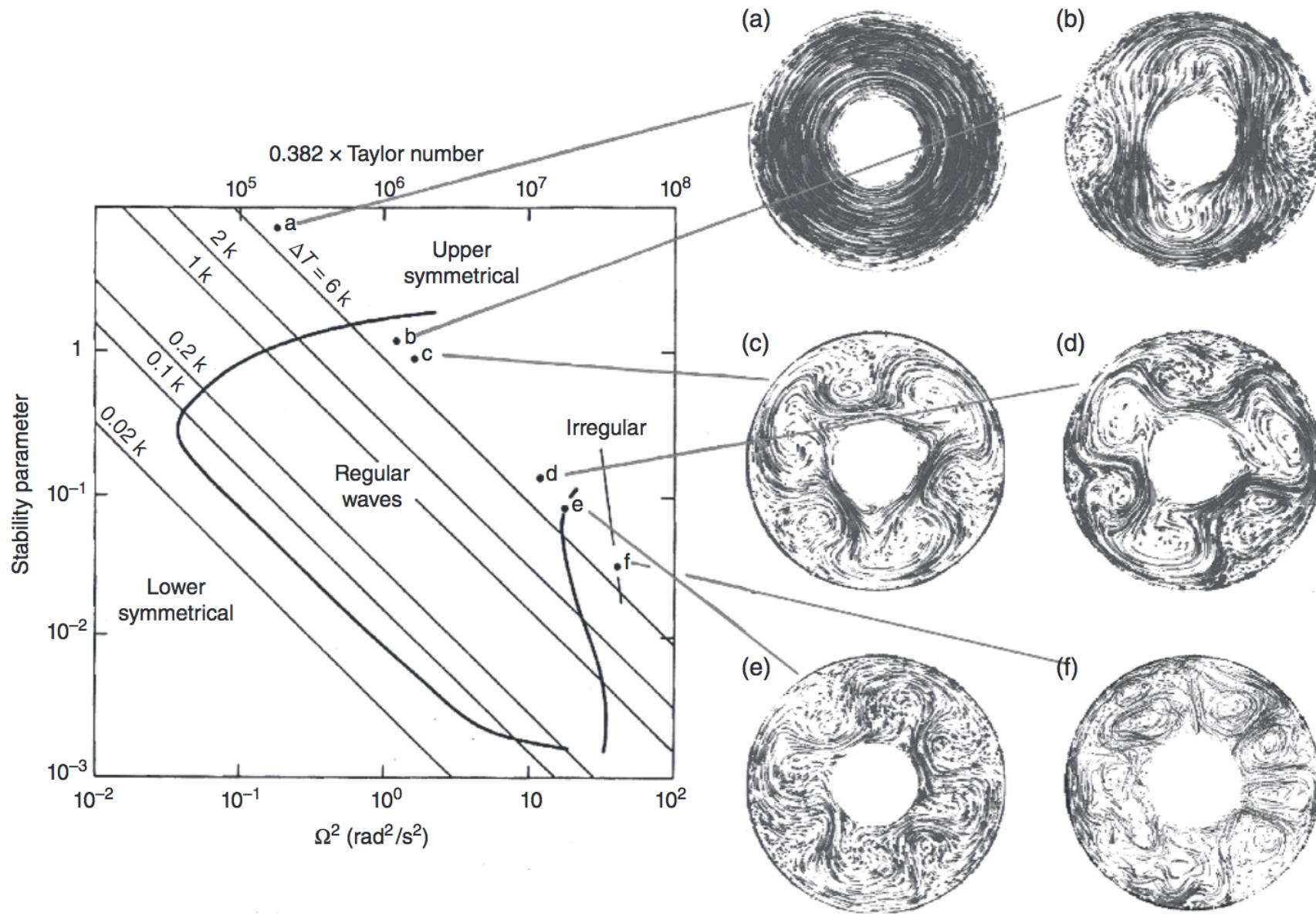
Figure 1.3. Schematic regime diagram for the thermally driven rotating annulus in relation to the thermal Rossby number Θ (or stability parameter, $\propto \Omega^{-2}$) and Taylor number $\mathcal{T} \propto \Omega^2$, showing some typical horizontal flow patterns at the top surface, visualized as streak images at upper levels of the experiment.

Read *et al.*, 2015
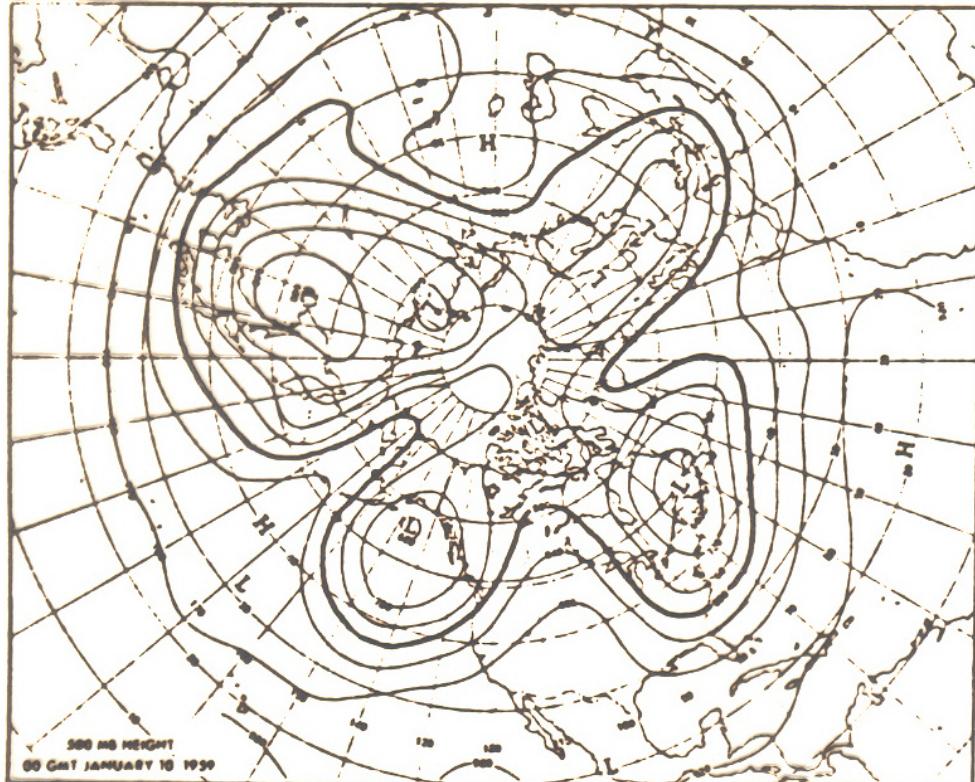
Figure 2. Comparison shows similarities between the global 500 mb pressure pattern in the upper atmosphere of the Northern Hemisphere and a four-wave pattern in the laboratory. (Laboratory flow conditions were similar to those in Fig. 1, except $\Omega = 1.95$ radians per sec.) In the atmosphere the flow is approximately parallel to the isobars (the flow is to the right, from high to low pressure), with speed inversely proportional to the spacing. Changes in the wave pattern have a significant effect on large-scale weather and climate.
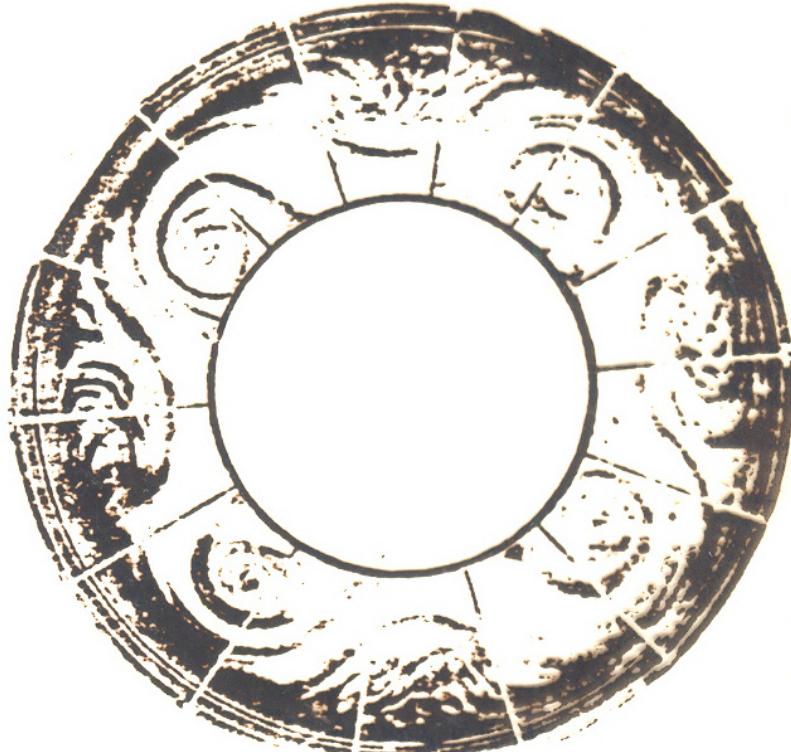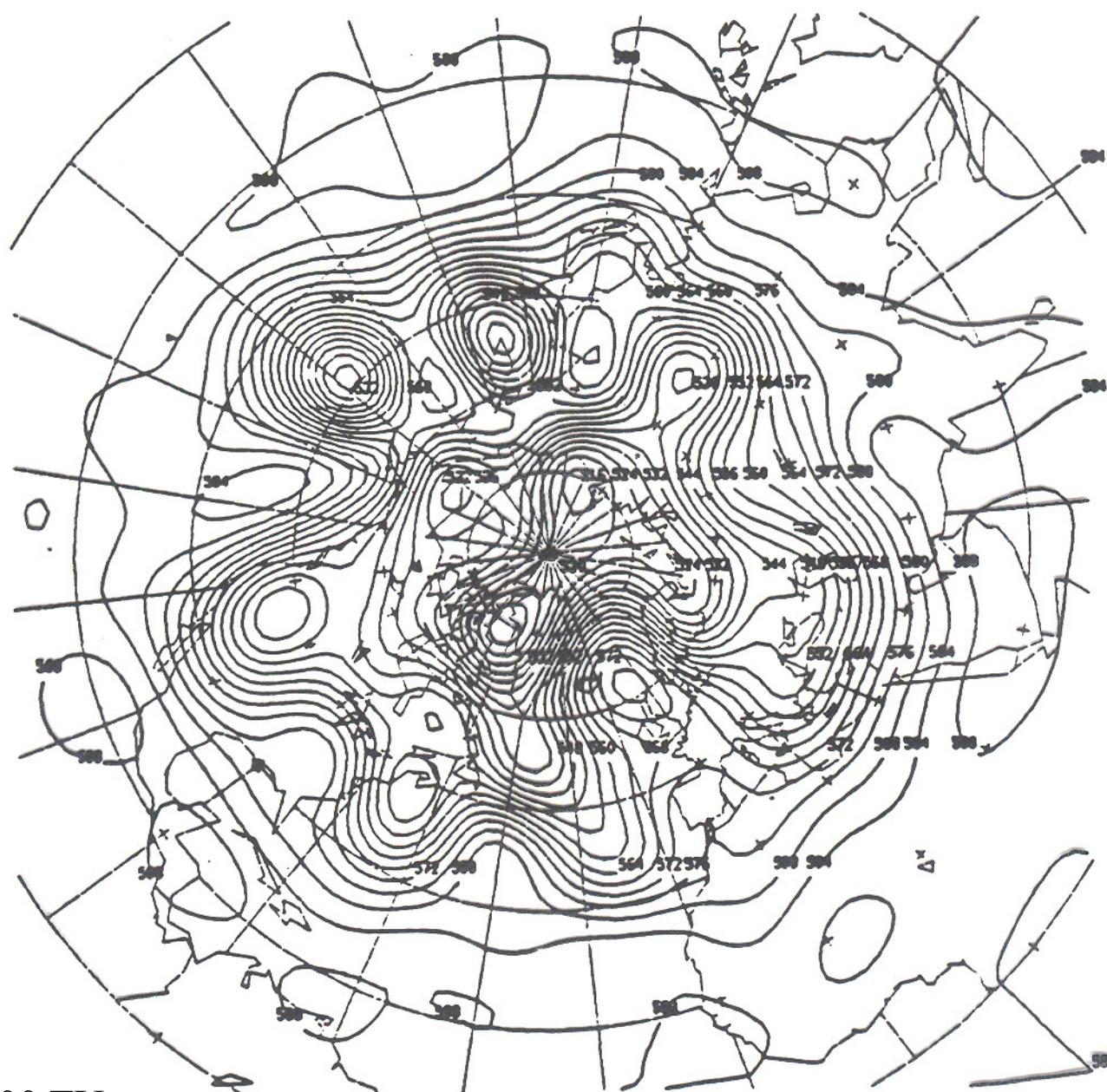
26/04/1984, 00/00 TU
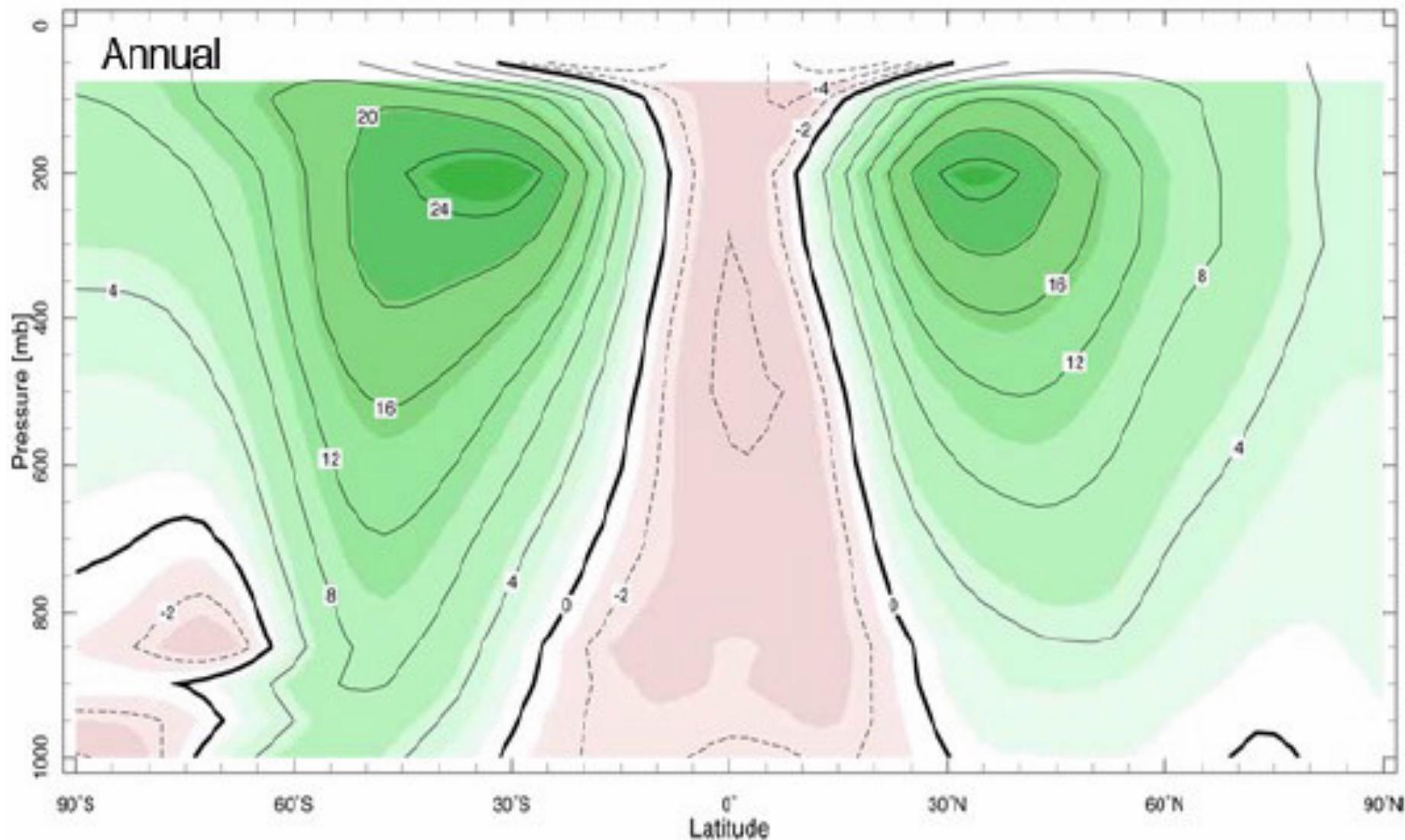
Zzonal wind; annual longitudinal average (m.s^{-1})
http://paoc.mit.edu/labweb/notes/chap5.pdf,
Atmosphere, Ocean and Climate Dynamics, by J. Marshall and R. A. Plumb,
International Geophysics, Elsevier)
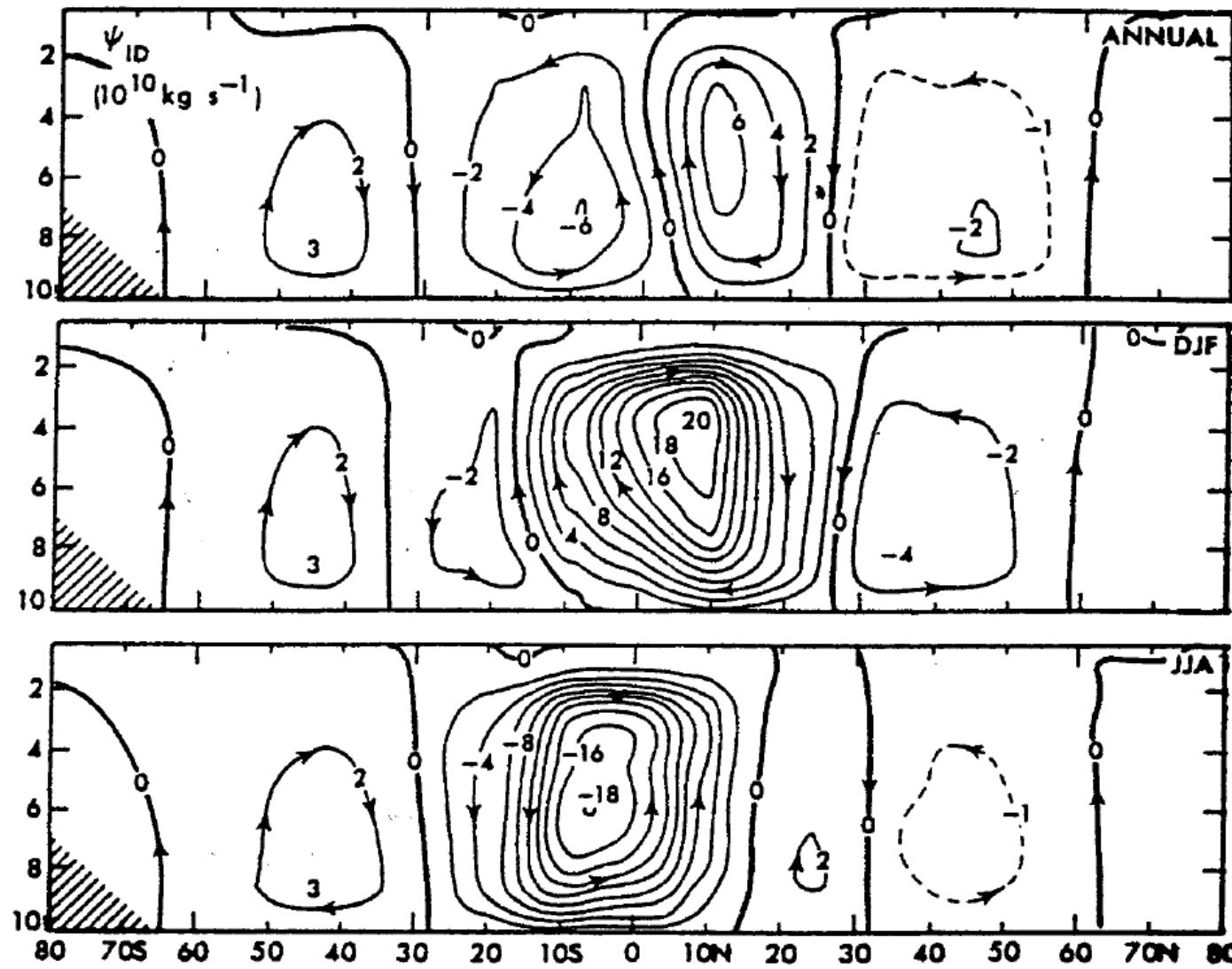
Peixoto and Oort, 1992, *The Physics of Climate,* Springer-Verlag

Base time: Sat 20 Mar 2021 00 UTC, Valid time: Thu 25 Mar 2021 00 UTC, - T+120 h, Area : Europe
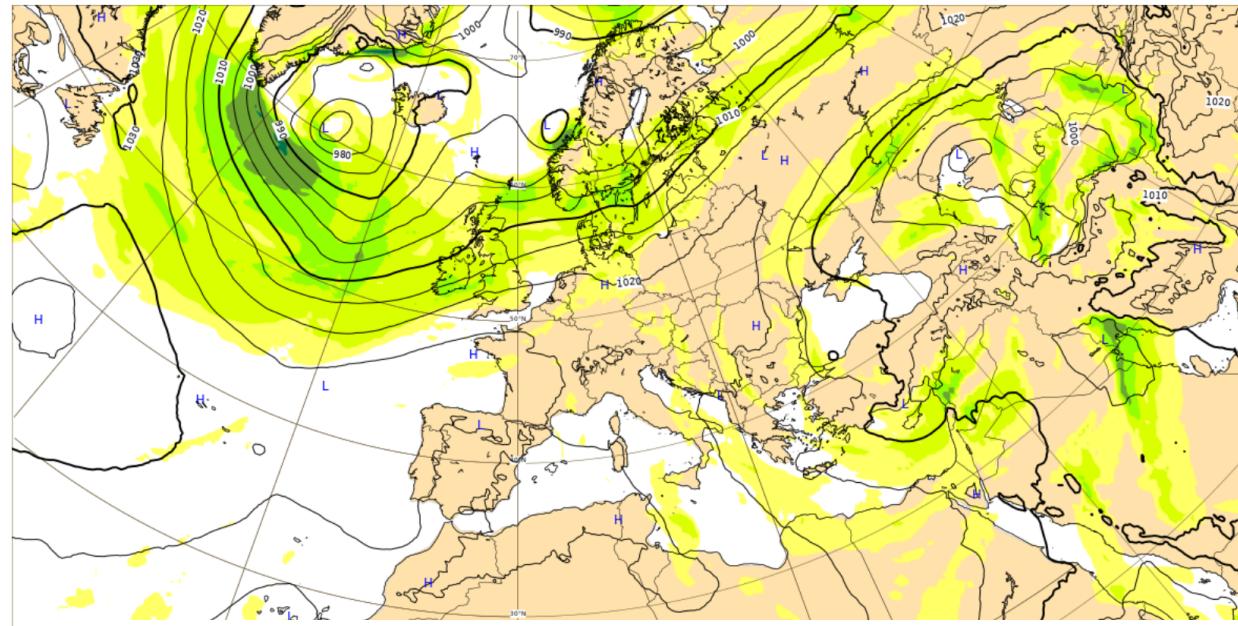
Base time: Thu 25 Mar 2021 00 UTC, Valid time: Thu 25 Mar 2021 00 UTC, - T+0 h, Area : Europe

850 hPa wind speed (ms**-1)

Mean sea level pressure (hPa)

100 (m/s)
80
60
50
40
30
25
20
15
10
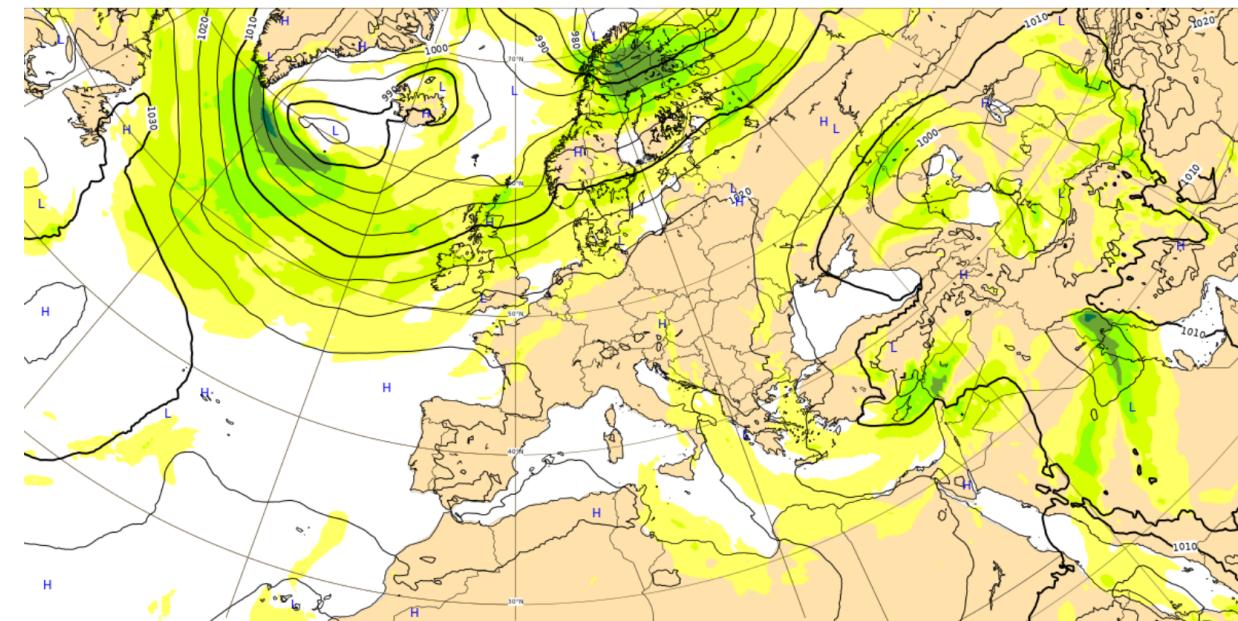
2021

Base time: Sat 20 Mar 2021 00 UTC, Valid time: Sat 20 Mar 2021 00 UTC, - T+0 h, Area : Europe

Base time: Thu 25 Mar 2021 00 UTC, Valid time: Thu 25 Mar 2021 00 UTC, - T+0 h, Area : Europe

850 hPa wind speed (ms**-1)

Mean sea level pressure (hPa)

2021

Control (Cluster 3)	Operations (Cluster 3)	Verifying Analysis	ECMWF ENSEMBLE FORECASTS

ECMWF ENSEMBLE FORECASTS
Monday   25 January 1993 12z ECMWF Forecast t+168 VT: Monday   1 February 1993 12z
500 hPa geopotential

Ensemble 1 (Cluster 1) | Ensemble 2 (Cluster 2) | Ensemble 3 (Cluster 1) | Ensemble 4 (Cluster 2) | Ensemble 5 (Cluster 3) | Ensemble 6 (Cluster 1) | Ensemble 7 (Cluster 2) | Ensemble 8 (Cluster 1)

Ensemble 9 (Cluster 2) | Ensemble 10 (Cluster 1) | Ensemble 11 (Cluster 1) | Ensemble 12 (Cluster 3) | Ensemble 13 (Cluster 1) | Ensemble 14 (Cluster 2) | Ensemble 15 (Cluster 1) | Ensemble 16 (Cluster 2)

Ensemble 17 (Cluster 3) | Ensemble 18 (Cluster 1) | Ensemble 19 (Cluster 3) | Ensemble 20 (Cluster 1) | Ensemble 21 (Cluster 2) | Ensemble 22 (Cluster 1) | Ensemble 23 (Cluster 1) | Ensemble 24 (Cluster 3)

Ensemble 25 (Cluster 1) | Ensemble 26 (Cluster 2) | Ensemble 27 (Cluster 3) | Ensemble 28 (Cluster 2) | Ensemble 29 (Cluster 3) | Ensemble 30 (Cluster 1) | Ensemble 31 (Cluster 3) | Ensemble 32 (Cluster 1)

Fig. 1: Members of day 7 forecast of 500 hPa geopotential height for the ensemble originated from
25 January 1993.
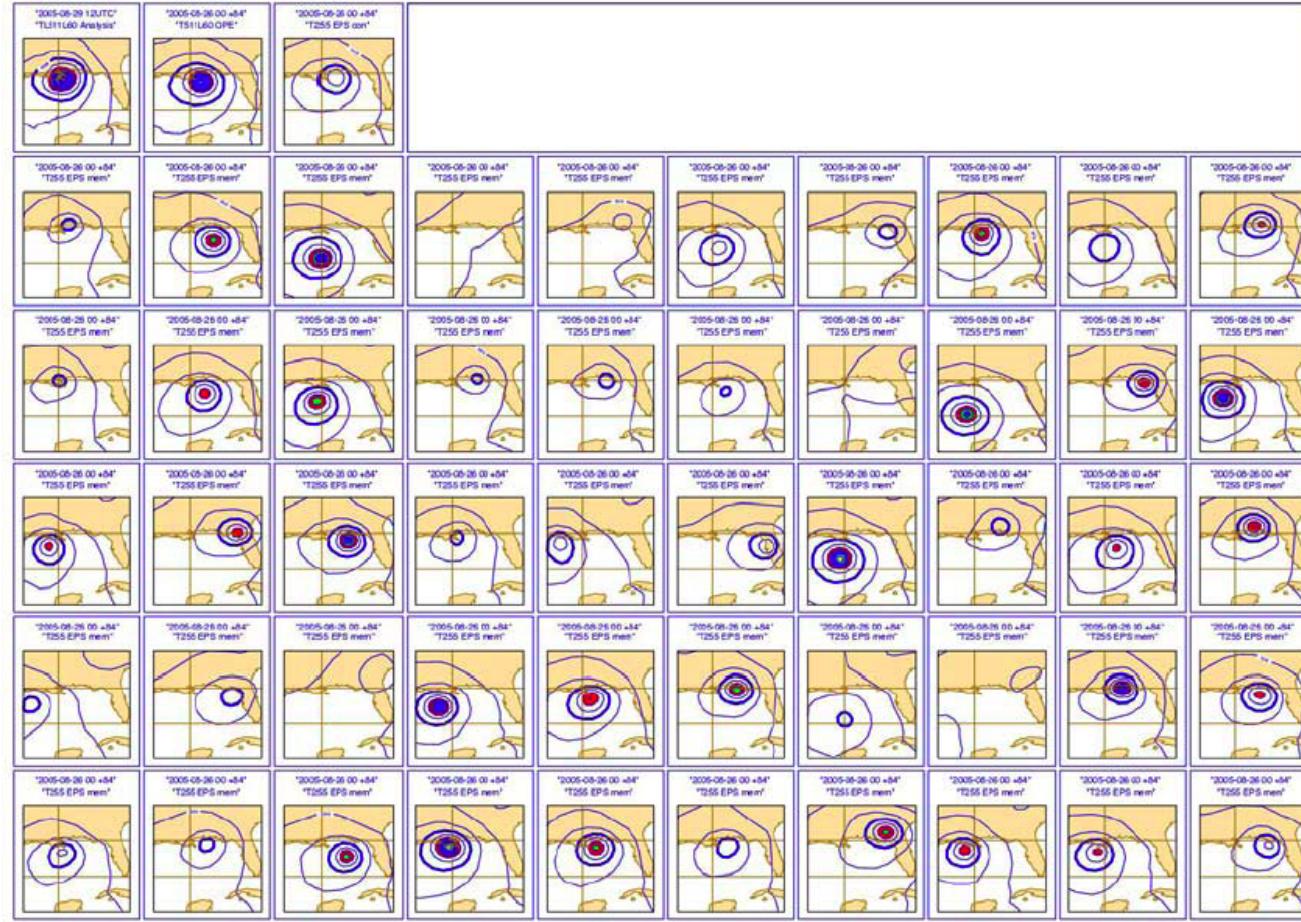
Figure 6 Hurricane Katrina mean-sea-level-pressure (MSLP) analysis for 12 UTC of 29 August 2005 and t+84h high-resolution and EPS forecasts started at 00 UTC of 26 August:

1st row: 1st panel: MSLP analysis for 12 UTC of 29 Aug
   *2nd panel: MSLP t+84h $T_L511L60$ forecast started at 00 UTC of 26 Aug*
    3rd panel: MSLP t+84h EPS-control $T_L255L40$ forecast started at 00 UTC of 26 Aug
Other rows: 50 EPS-perturbed $T_L255L40$ forecast started at 00 UTC of 26 Aug.

The contour interval is 5 hPa, with shading patters for MSLP values lower than 990 hPa.

ECMWF, Technical Report 499, 2006

ECMWF ENSEMBLE FORECASTS
Monday 15 March 2021 0000 UTC ECMWF forecast t+168 VT:Monday 22 March 2021 0000 UTC
MSLP (contour every 5hPa) Temperature at 850hPa (only -6 and 16 isolines are plotted)
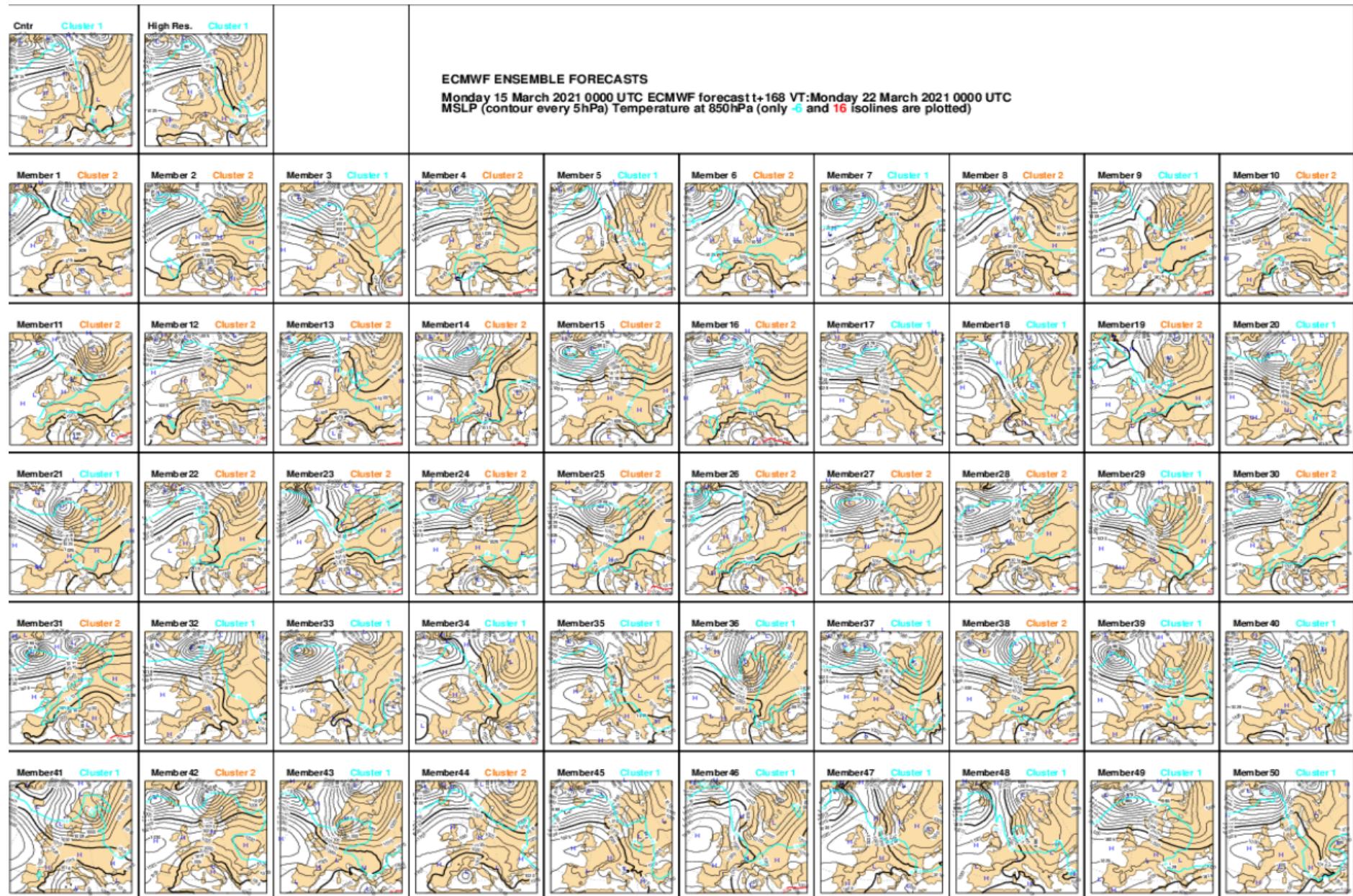
Pourquoi les météorologistes ont-ils tant de peine à prédire le temps avec quelque certitude ? Pourquoi les chutes de pluie, les tempêtes elles-mêmes nous semblent-elles arriver au hasard, de sorte que bien des gens trouvent tout naturel de prier pour avoir la pluie ou le beau temps, alors qu'ils jugeraient ridicule de demander une éclipse par une prière ? Nous voyons que les grandes perturbations se produisent généralement dans les régions où l'atmosphère est en équilibre instable. Les météorologistes voient bien que cet équilibre est instable, qu'un cyclone va naître quelque part ; mais où, ils sont hors d'état de le dire ; un dixième de degré en plus ou en moins en un point quelconque, le cyclone éclate ici et non pas là, et il étend ses ravages sur des contrées qu'il aurait épargnées. Si on avait connu ce dixième de degré, on aurait pu le savoir d'avance, mais les observations n'étaient ni assez serrées, ni assez précises, et c'est pour cela que tout semble dû à l'intervention du hasard.

H. Poincaré, *Science et Méthode*, Paris, 1908

Why have meteorologists such difficulty in predicting the weather with any certainty? Why is it that showers and even storms seem to come by chance, so that many people think it quite natural to pray for rain or fine weather, though they would consider it ridiculous to ask for an eclipse by prayer? We see that great disturbances are generally produced in regions where the atmosphere is in unstable equilibrium. The meteorologists see very well that the equilibrium is unstable, that a cyclone will be formed somewhere, but exactly where they are not in a position to say; a tenth of a degree more or less at any given point, and the cyclone will burst here and not there, and extend its ravages over districts it would otherwise have spared. If they had been aware of this tenth of a degree they could have known it beforehand, but the observations were neither sufficiently comprehensive nor sufficiently precise, and that is the reason why it all seems due to the intervention of chance.

H. Poincaré, *Science et Méthode*, Paris, 1908

(English transl. by F. Maitland, *Science and Method*,

T. Nelson and Sons, London, 1914)

Physical laws governing the flow

- Conservation of mass

  $D\rho/Dt + \rho \, \mathrm{div}\underline{U} = 0$

- Conservation of energy

  $De/Dt - (p/\rho^2)\, D\rho/Dt = Q$

- Conservation of momentum

  $D\underline{U}/Dt + (1/\rho)\, \mathrm{grad}p - \underline{g} + 2\, \underline{\Omega} \wedge \underline{U} = \underline{F}$

- Equation of state

  $f(p, \rho, e) = 0$ $\qquad\qquad\qquad (p/\rho = rT, e = C_v T)$

- Conservation of mass of secondary components (water, chemical species, …)

  $Dq/Dt + q\, \mathrm{div}\underline{U} = S$

These physical laws must be expressed in practice in discretized (and necessarily imperfect) form, both in space and time

22

The case of the ocean

- Same basic equations

But

- Different equation of state

- Major secondary component is now salt (convective instability associated with variation of density of the fluid)

Physical laws must in practice be discretized in both space and time
⇒ *numerical models*, which are necessarily imperfect.

Models that are used for large scale weather prediction and for climatological
   simulation cover the whole volume of the atmosphere. These models are
   based, at least so far, on the *hydrostatic* hypothesis

   in the vertical direction :

   $\partial p/\partial z + \rho g = 0$

   Eliminates momentum equation for vertical direction. In addition, flow is
   incompressible in coordinates (x, y, p) ⇒ number of equations decreased by
   two units.

   Hydrostatic approximation valid, to accuracy $\approx 10^{-4}$, for horizontal scales
   > 20-30 km

   More costly nonhydrostatic models are used for small scale meteorology.

Hydrostatic approximation allows to take pressure p as independent vertical coordinate

- Flow is incompressible

- Pressure gradient term $(1/\rho)\ \mathrm{grad}_z\ p$ becomes $\mathrm{grad}_p\ \Phi$, where $\Phi \equiv gz$ is geopotential

Spatial Discretization

There exist at present two forms of spatial discretization

- Gridpoint discretization

- (Semi-)spectral discretization (mostly for global models, and most often only in the horizontal direction)

Finite element discretization, which is very common in many forms of numerical modelling, is rarely used for modelling of the atmosphere, except for discretization in the vertical direction. It is more frequently used for oceanic modelling, where it allows to take into account the complicated geometry of coast-lines.

In *gridpoint models*, meteorological fields are defined by values at the nodes of a grid covering the physical domain under consideration. Spatial and temporal derivatives are expressed by finite differences.

A schematic of an Atmospheric General Circulation Model (L. Fairhead /LMD-CNRS)
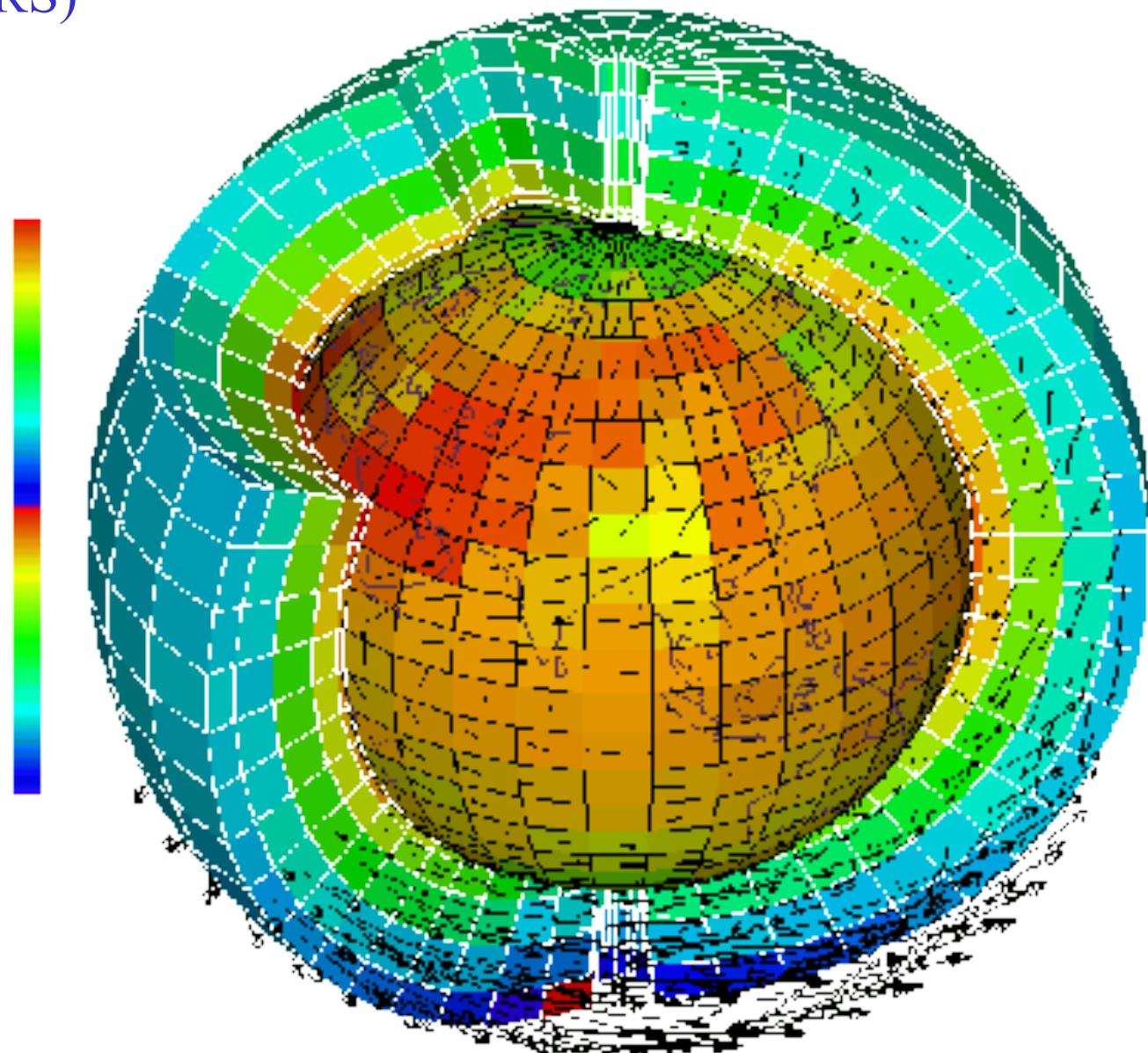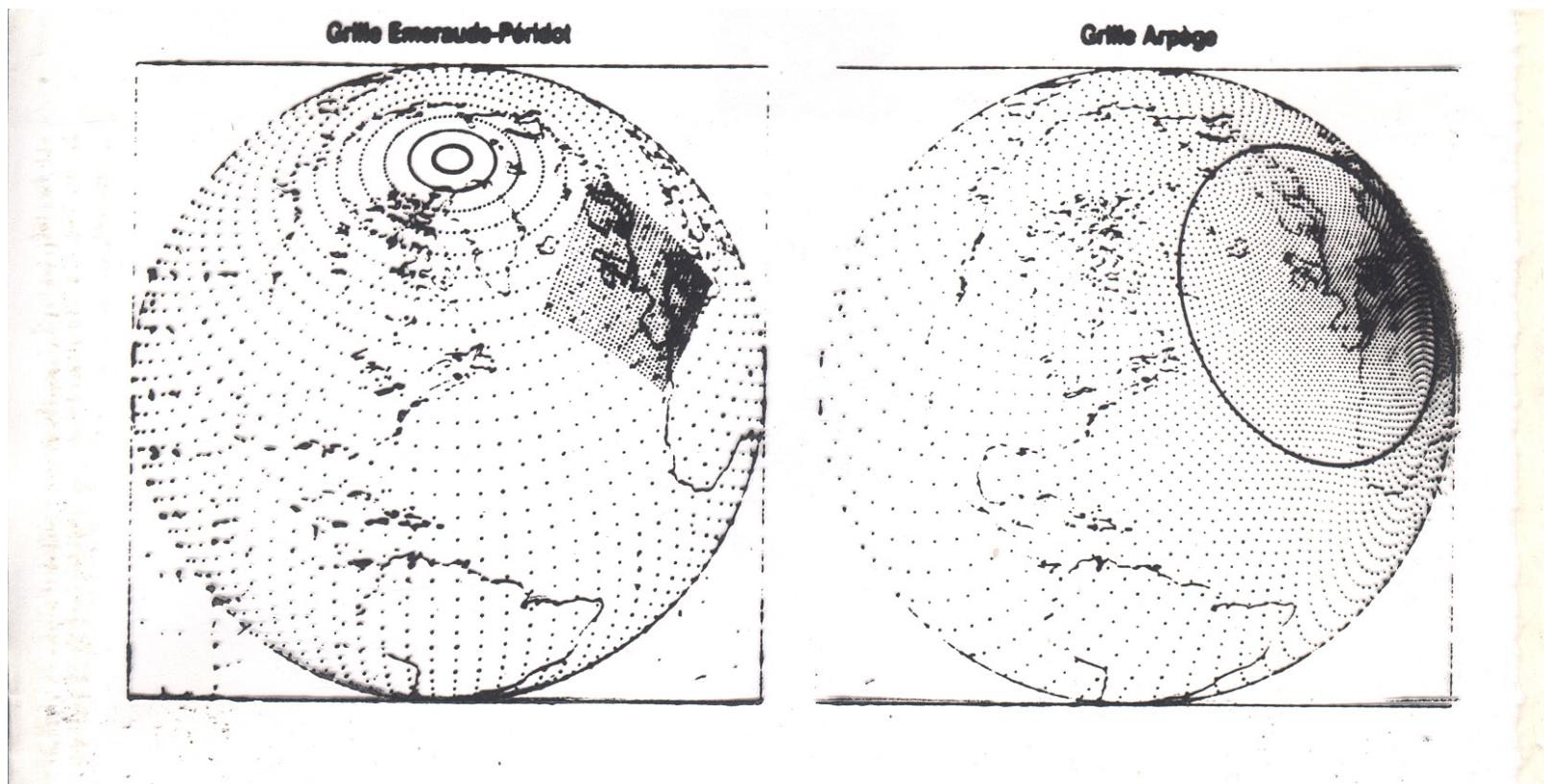
Grilles de modèles de Météo-France *(La Météorologie)*

In *spectral models*, fields are defined by the coefficients of their expansion along a prescribed set of basic functions. In the case of global meteorological models, those basic functions are the spherical harmonics (eigenfunctions of the laplacian at the surface of the sphere).

Modèles (semi-)spectraux

$$T(\mu=\sin(\text{latitude}),\ \lambda=\text{longitude}) = \sum_{\substack{0 \le n < \infty \\ -n \le m \le n}} T_n^m Y_n^m(\mu,\lambda)$$

où les $Y_n^m(\mu,\lambda)$ sont les *harmoniques sphériques*

$$Y_n^m(\mu,\lambda) \propto P_n^m(\mu)\exp(im\lambda)$$

$P_n^m(\mu)$ est la *fonction de Legendre* de deuxième espèce

$$P_n^m(\mu) \propto (1-\mu^2)^{\frac{m}{2}} \frac{d^{n+m}}{d\mu^{n+m}}(\mu^2-1)^n$$

n et m sont respectivement le *degré* et l'*ordre* de l'harmonique $Y_n^m(\mu,\lambda)$

$n = 0, 1, \ldots$ $-n \le m \le n$
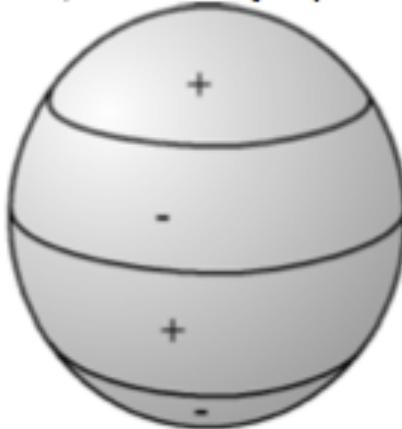
$l = 3$
$m = 0$
$l-m = 3$

$l = 3$
$m = 1$
$l-m = 2$

$l = 3$
$m = 2$
$l-m = 1$

$l = 3$
$m = 3$
$l-m = 0$

$l = 5$
$m = 2$
$l-m = 3$
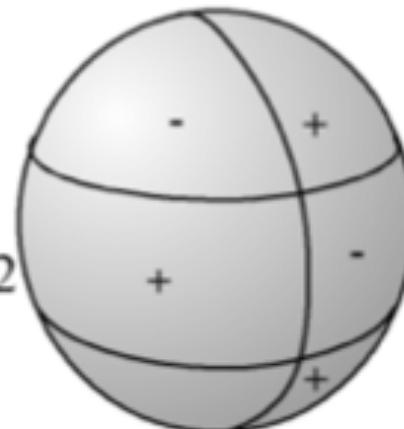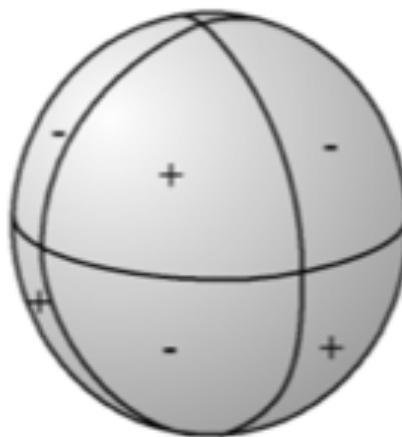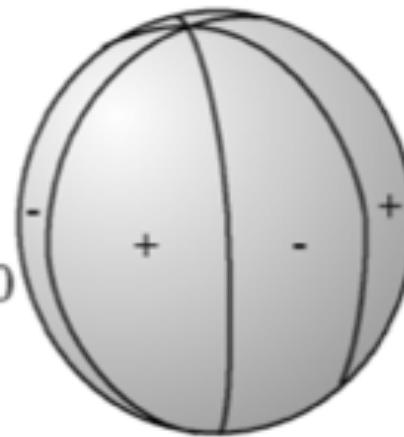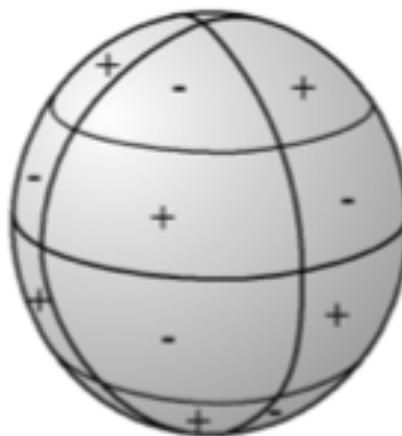
Modèles (semi-)spectraux

Les harmoniques sphériques définissent une base complète orthonormée de l'espace L^2 à la surface S de la sphère.

$$\int_S Y_n^m [Y_{n'}^{m'}]^* d\mu d\lambda = \delta_n^{n'} \delta_m^{m'}$$

Relation de Parseval

$$\int_S T^2(\mu,\lambda) d\mu d\lambda = \sum_{\substack{0 \leq n < \infty \\ -n \leq m \leq n}} \left| T_n^m \right|^2$$

Les harmoniques sphériques sont fonctions propres du laplacien à la surface de la sphère

$$\Delta Y_n^m = -n(n+1)Y_n^m$$

Troncature 'triangulaire' *TN* ($n \leq N$, $-n \leq m \leq n$) indépendante du choix d'un axe polaire. Représentation est parfaitement homogène à la surface de la sphère

Calculs non linéaires effectués dans l'espace physique (sur grille appropriée, souvent latitude-longitude 'gaussienne'). Les transformations requises sont possibles à un coût non prohibitif grâce à l'utilisation de Transformées de Fourier Rapides (*Fast Fourier Transforms*, *FFT*, en anglais). Il existe aussi une version rapide des Transformées de Legendre, relatives à la variable μ.

In addition to hydrostatic approximation, the following approximations are (almost) systematically made in global modeling :

- Atmospheric fluid is contained in a spherical shell with negligible thickness. This does not forbid the existence within the shell of a vertical coordinate which, in view of the hydrostatic equation, can be chosen as the pressure p.

- The horizontal component of the Coriolis acceleration due to the vertical motion is neglected (this approximation, sometimes called the *traditional approximation*, is actually a consequence of the previous one).

- Tidal forces are neglected.

These approximations lead to the so-called (and ill-named) *primitive equations*

Pressure p, although convenient for writing down the equations, is in fact rather inconvenient because lower boundary is not fixed in (x, y, p)-space.

So-called σ-coordinate. $\sigma \equiv p/p_S$, where p_S is pressure at ground level.

'Hybrid' coordinate.

Physical laws governing the flow

- Conservation of mass

$$D\rho/Dt + \rho \, \mathrm{div}\underline{U} = 0$$

- Conservation of energy

$$De/Dt - (p/\rho^2) \, D\rho/Dt = Q$$

- Conservation of momentum

$$D\underline{U}/Dt + (1/\rho) \, \mathrm{grad}p - \underline{g} + 2\,\underline{\Omega} \wedge \underline{U} = \underline{F}$$

- Equation of state

$$f(p, \rho, e) = 0 \qquad\qquad (p/\rho = rT, \, e = C_v T)$$

- Conservation of mass of secondary components (water in the atmosphere, salt in the ocean, chemical species, …)

$$Dq/Dt + q \, \mathrm{div}\underline{U} = S$$

These physical laws must be expressed in practice in discretized (and necessarily imperfect) form, both in space and time

37

Parlance of the trade :

- Adiabatic and inviscid, and therefore thermodynamically reversible, processes (everything except Q, F and S) make up *'dynamics'*

- Processes described by terms Q, F and S make up *'physics'*

As said before, one ordinarily distinguishes two different parts in models. The 'dynamics' deals with the physically reversible processes (pressure forces, Coriolis force, advection, …), while the 'physics' deals with physically irreversible processes, in particular the diabatic heating term Q in the energy equation, and also the parameterization of subgrid scales effects.

Numerical schemes have been gradually developed and validated for the 'dynamics' component of models, which are by and large considered now to work satisfactorily (although regular improvements are still being made; project *DYNAMICO*, *Dynamical Core on Icosahedral Grid*, Th. Dubos, IPSL).

The situation is different as concerns 'physics', where many problems remain (as concerns for instance subgrid scales parameterization, the water cycle and the associated exchanges of energy, or the exchanges that take place in the boundary layer between the atmosphere and the underlying medium). 'Physics' as a whole remains the weaker point of models, and is still the object of active research.

5 - SCHEMA DES INTERACTIONS PHYSIQUES DANS LE MODELE
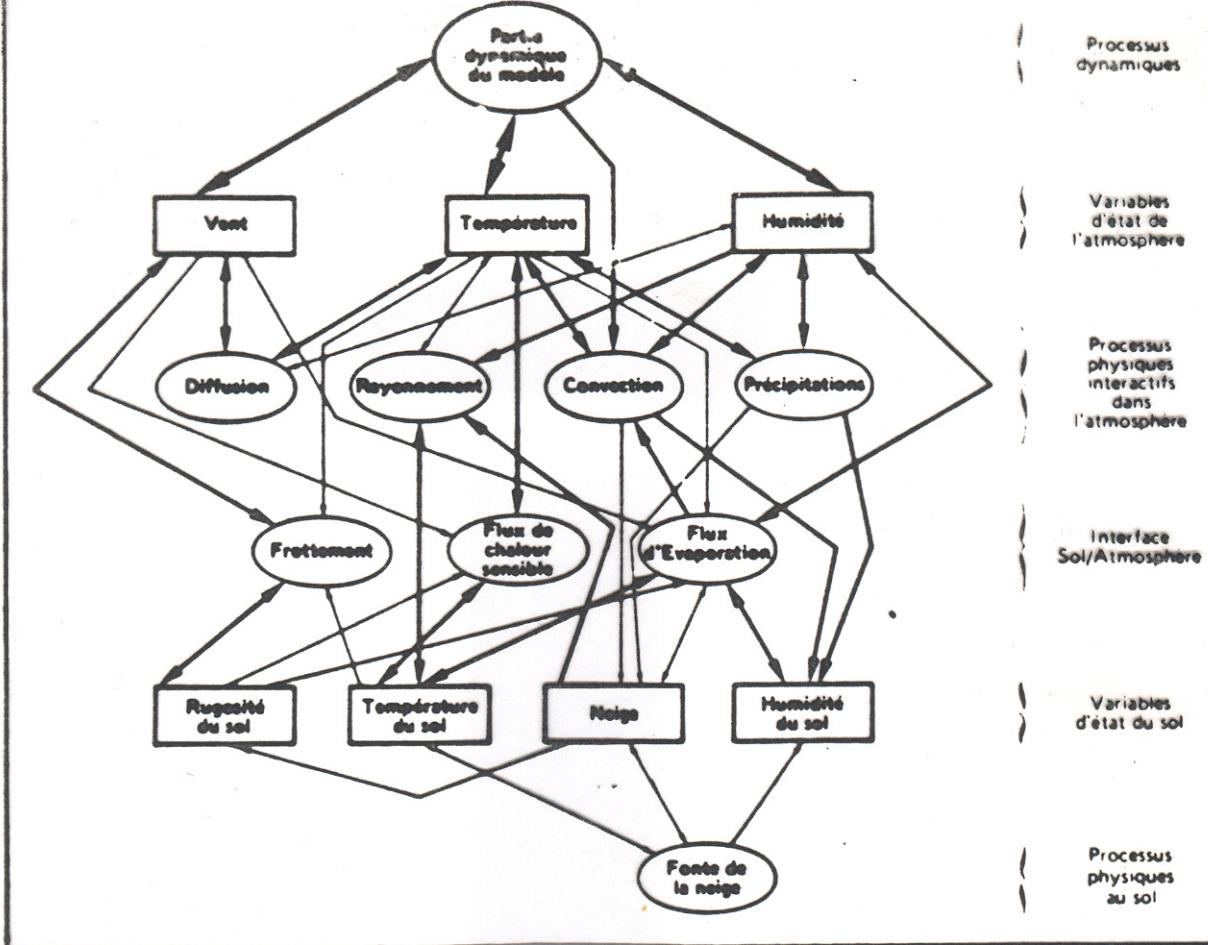
Cours à venir

~~Vendredi 26 mars~~

Vendredi 2 avril

Vendredi 9 avril

Vendredi 16 avril

Vendredi 7 mai

Vendredi 14 mai

Vendredi 21 mai

Vendredi 28 mai