Atmospheric dynamics and meteorology

B. Legras, http://www.lmd.ens.fr/legras

III Frontogenesis

(pre requisite: quasi-geostrophic equation, baroclinic instability in the Eady and Phillips models)

Recommended books:

- Holton, An introduction to dynamic meteorology, Academic Press
- Houze, Cloud dynamics, Academic Press
- Vallis, Atmospheric and oceanic fluid dynamics

Atmospheric circulation from the viewpoint of a geograph

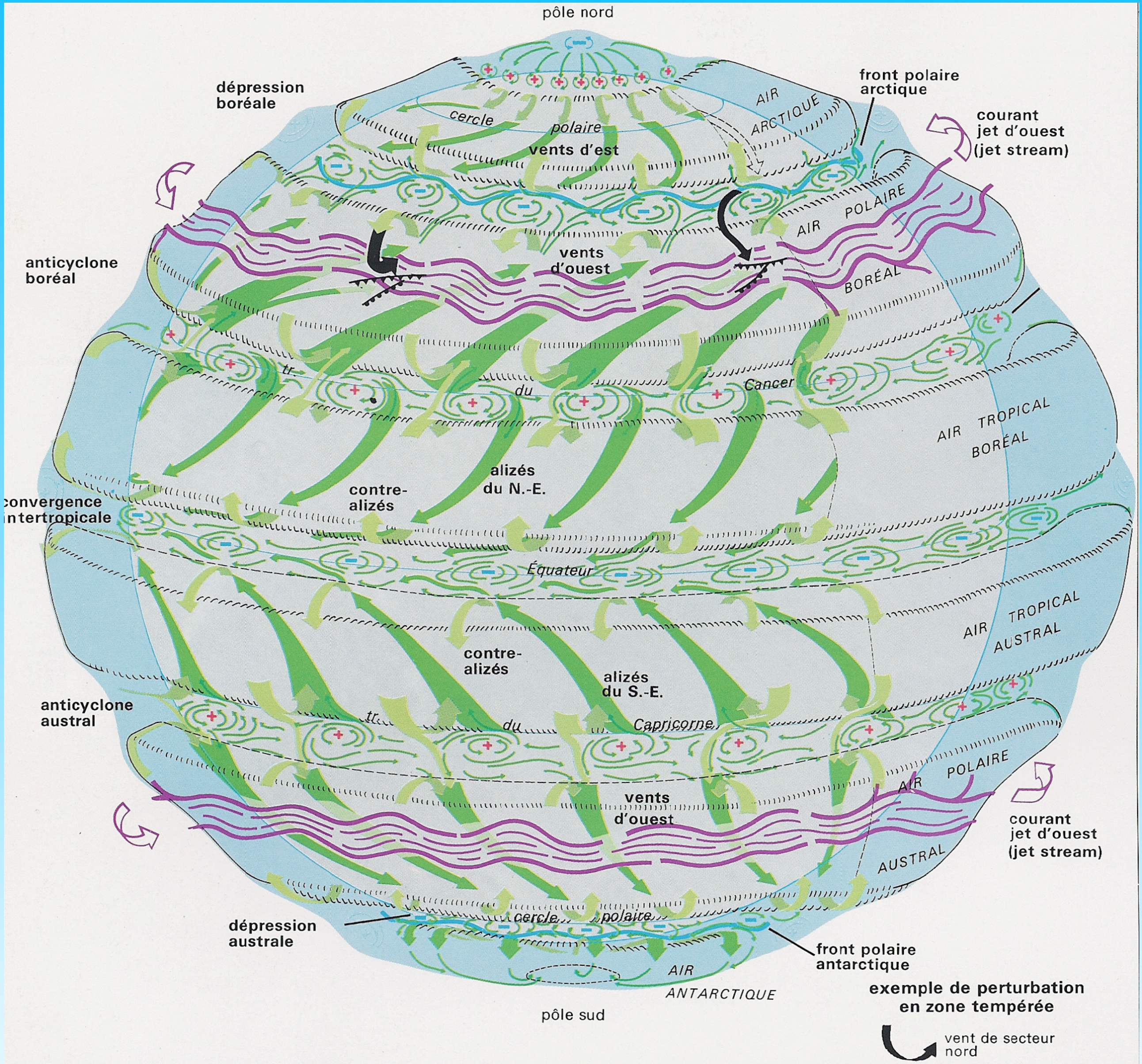


schéma de la circulation générale dans la basse atmosphère

le croquis ci-dessus n'est qu'un schéma, en particulier il ne tient pas compte de la présence d'océans ou de continents ni de l'alternance des saisons.

Frontogenesis

- Boussinesq equations
- Quasi-geostrophic frontogenesis
- Semi-geostrophic frontogenesis
- Influence of moisture
- Large-scale front
- Frontogenesis and storms

EQUATIONS

$$D_t u - f\,v + \partial_x \phi' = 0$$
$$D_t v + f\,u + \partial_y \phi' = 0$$
$$-b' + \partial_{\tilde z} \phi' = 0$$
$$D_t b = D_t b' + \tilde w N^2 = 0$$
$$\partial_x u + \partial_y v + \partial_{\tilde z} \tilde w = 0$$

with $b' = \frac{g}{\theta_0}\theta' = \frac{\partial \phi'}{\partial \bar z}$

(buoyancy),

$\tilde w = D_t \tilde z$,

and $N^2 = \frac{g}{\theta_0} d_{\tilde z} \bar\theta$

FROM HYDROSTATIC EQUATIONS IN PRESSURE COORDINATE

The standard air profile is defined by $\bar\theta$ and $\bar\phi$ (geopotential) which are function of p only.

ϕ' is the geopotential deviation: $\phi' = \phi - \bar\phi$.

A new form of the pressure coordinate is introduced as a pseudo-height by $\tilde z = [1 - (\frac{p}{p_0})^\kappa]\frac{H_0}{\kappa}$ where $H_0 = R\theta_0/g$.

This coordinate is such that

$$d\tilde z = \frac{-dp}{p}\left(\frac{p}{p_0}\right)^\kappa H_0 = \frac{g\,dz}{RT}\left(\frac{p}{p_0}\right)^\kappa H_0 = \frac{\theta_0}{\theta}dz$$

(for an adiabatic atmosphere, one would have exactly $\tilde z = z$)

The hydrostatic equation is now $\frac{\partial g z}{\partial \tilde z} = g\frac{\theta}{\theta_0}$

Since $\omega = D_t p = -\frac{p}{H_0}\left(\frac{p_0}{p}\right)^\kappa D_t \tilde z$, we have

$$\frac{\partial \omega}{\partial p} = \partial_p D_t p = \partial_{\tilde z}\tilde w - \frac{\tilde w}{\bar H} \quad \text{with } \tilde w = D_t \tilde z \text{ and } \bar H = \frac{H_0}{1-\kappa}\left(\frac{p}{p_0}\right)^\kappa$$

The Boussinesq approximation consists here in neglecting the term in $\frac{\tilde w}{\bar H}$

REMINDER: Derivation of the quasi-geostrophic equation in a channel flow $f = f_0$ constant

The horizontal flow can be separated into its geostrophic and ageostrophic component as $u = u_g + u_a$ and $v = v_g + v_a$ where $f_0 u_g = \frac{-\partial \phi'}{\partial y}$ and $f_0 v_g = \frac{\partial \phi'}{\partial x}$.

The quasi-geostrophic approximation is based on the assumption that $u_a \ll u_g$ and $v_a \ll v_g$. More precisely the ratio between ageostrophic and geostrophic motion is of the order of the Rossby number .

Then the advection is reduced to $D_{gt} = \frac{\partial}{\partial t} + u_g \frac{\partial}{\partial x} + v_g \frac{\partial}{\partial y}$ and the horizontal motion obeys $D_{gt} u_g - f_0 v_a = 0$ and $D_{gt} v_g + f_0 u_a = 0$.

The buoyancy equation is reduced to $D_{gt} b' + w N^2 = 0$.

The equation for the geostrophic vorticity $\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}$ is $D_{gt} \zeta_g + f_0 \left(\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} \right) = 0$

Derivating the buoyancy equation by z after dividing it by N^2 and using the continuity equation $\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} + \frac{\partial w}{\partial z} = 0$,

we get $D_{gt} \left(\zeta_g + \frac{\partial}{\partial z} \left(\frac{f_0}{N^2} b' \right) \right) = 0$.

Now we use $f_0 \zeta_g = \frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \phi'}{\partial y^2}$ and $b' = \frac{\partial \phi}{\partial z}$, to obtain

$$D_{gt} q = 0 \quad \text{with} \quad q = \frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \phi'}{\partial y^2} + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \frac{\partial \phi'}{\partial z} \right)$$

This is the quasi-geostrophic equation.
Notice that we have dropped the tilde on z for simplicity.
Notice also that if the ageostrophic flow is absent in the final equation, it determines the evolution of the geostrophic flow. All the dynamics of the quasi-geostrophic model is in the "quasi".

The Rossby $\beta \frac{\partial \phi'}{\partial x}$ term in q can be obtained by changing the horizontal equation into

$$D_{gt} u_g - f_0 v_a - \beta y v_g = 0$$
$$D_{gt} v_g + f_0 u_a + \beta y u_g = 0$$

Frontogenesis

- Boussinesq equations
- Quasi-geostrophic frontogenesis
- Semi-geostrophic frontogenesis
- Influence of moisture
- Large-scale front
- Frontogenesis and storms

Quasi geostrophic frontogenesis

Quasi-geostrophic form of the Boussinesq equations

The geostrophic and non geostrophic parts are separated

$$u = u_g + u_a \quad \text{and} \quad v = v_g + v_a$$

with $f_0 u_g = -\partial_y \phi$ and $f_0 v_g = \partial_x \phi$

Hence

$$D_{gt} u_g - f_0 v_a = 0$$
$$D_{gt} v_g + f_0 u_a = 0$$
$$D_{gt} b + w N^2 = 0$$
$$\partial_x u_a + \partial_y v_a + \partial_z w = 0$$

with $D_{gt} \equiv \partial_t + u_g \partial_x + v_g \partial_y$

THERMAL WIND

$$f_0 \partial_z u_g = -\partial_y b$$
$$f_0 \partial_z v_g = \partial_x b$$

$$D_{gt} \partial_x b = Q_1 - N^2 \partial_x w$$
$$D_{gt} f_0 \partial_z v_g = -Q_1 - f_0^2 \partial_z u_a$$

where $Q_1 = -\partial_x u_g \partial_x b - \partial_x v_g \partial_y b$

We have $2 Q_1 = N^2 \partial_x w - f_0^2 \partial_z u_a$

Assuming that the geostrophic wind is oriented along y with small variations in this direction $\partial_y \ll \partial_x, \partial_z$. $u_g = 0, \partial_y v_a = 0$

Let define $u_a = \partial_z \psi$ et $w = -\partial_x \psi$, hence

$$-2 Q_1 = N^2 \partial_{xx}^2 \psi + f_0^2 \partial_{zz}^2 \psi$$

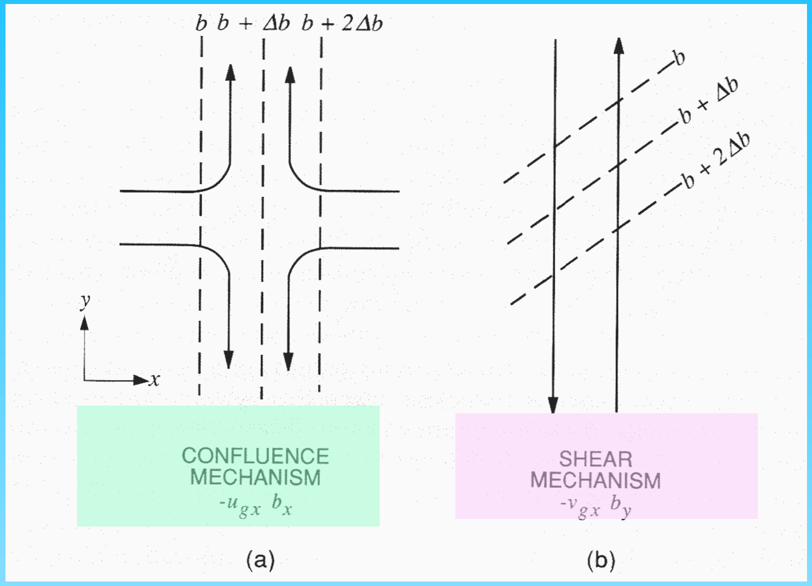
Apparent paradox:

Q_1 is associated to the geostrophic circulation

It destroys the thermal wind balance.

It is restored by the non geostrophic wind

Q_1 intensifies the temperature gradient by two mechanisms: confluence ($\partial_x u_g<0$) and gradient shear ($\partial_x v_g \partial_y b \neq 0$)

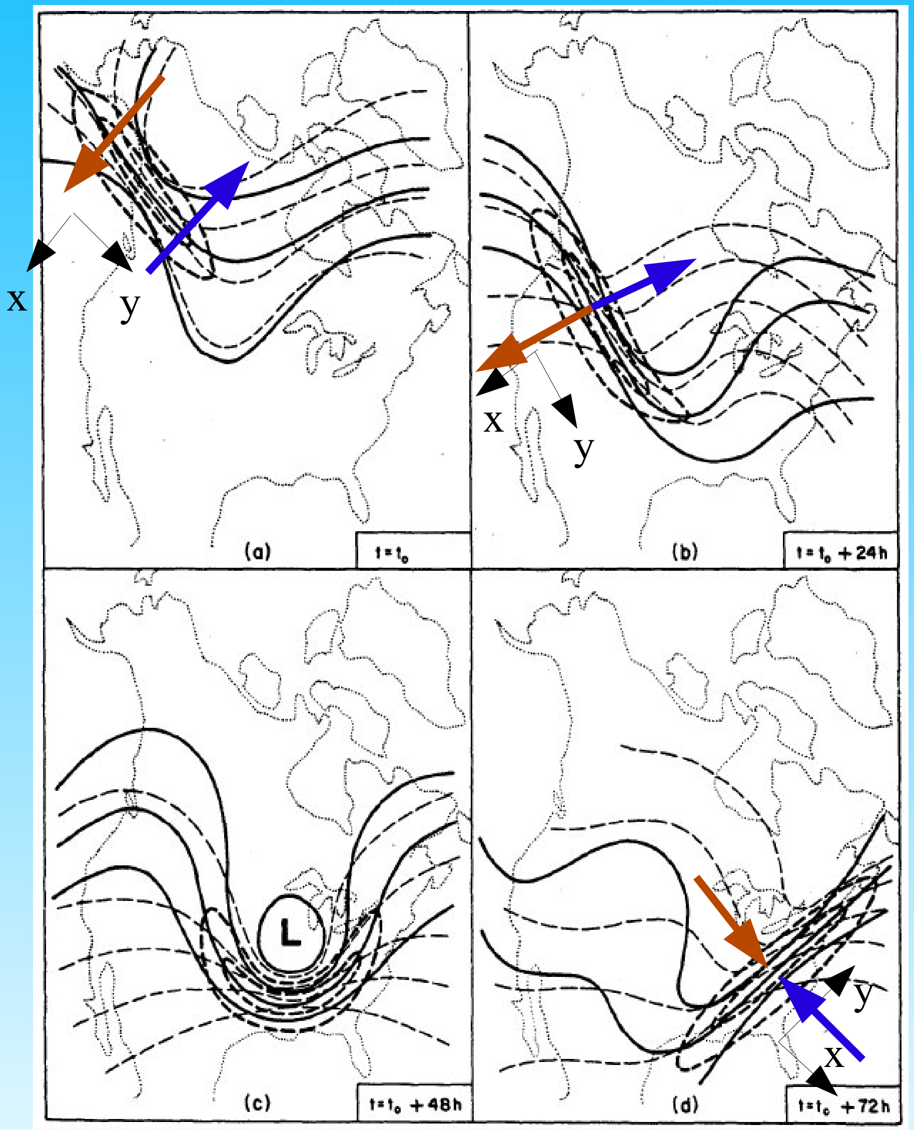


Mechanisms of gradient amplification.

$$D_{gt}\partial_x b = Q_1 - N^2 \partial_x w$$
$$\text{où } Q_1 = -\partial_x u_g \partial_x b - \partial_x v_g \partial_y b$$

Confluence

Cold advection



Warm advection

Q1 frontogenetic

Q1 frontolytic

More generally the temporal evolution of the thermal wind

$$f_0 \partial_z v_g = \partial_x b$$
$$f_0 \partial_z u_g = -\partial_y b$$

gives

$$D_{gt} \partial_x b = Q_1 - N^2 \partial_x w$$
$$D_{gt} f \partial_z v_g = -Q_1 - f^2 \partial_z u_a$$

$$D_{gt} \partial_y b = Q_2 - N^2 \partial_y w$$
$$-D_{gt} f_0 \partial_z u_g = -Q_2 - f_0^2 \partial_z v_a$$

where

$$Q_1 = -\partial_x u_g \partial_x b - \partial_x v_g \partial_y b$$
$$Q_2 = -\partial_y u_g \partial_x b - \partial_y v_g \partial_y b$$

Let denote $\vec{\boldsymbol{Q}} = (Q_1, Q_2)$

Generation of the ageostrophic circulation

$$2\vec{\boldsymbol{Q}} = N^2 \nabla_H w - f^2 \partial_z \vec{\boldsymbol{u}}_{\boldsymbol{a}}$$

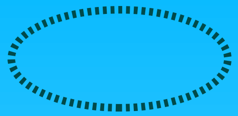
in other terms (omega equation)

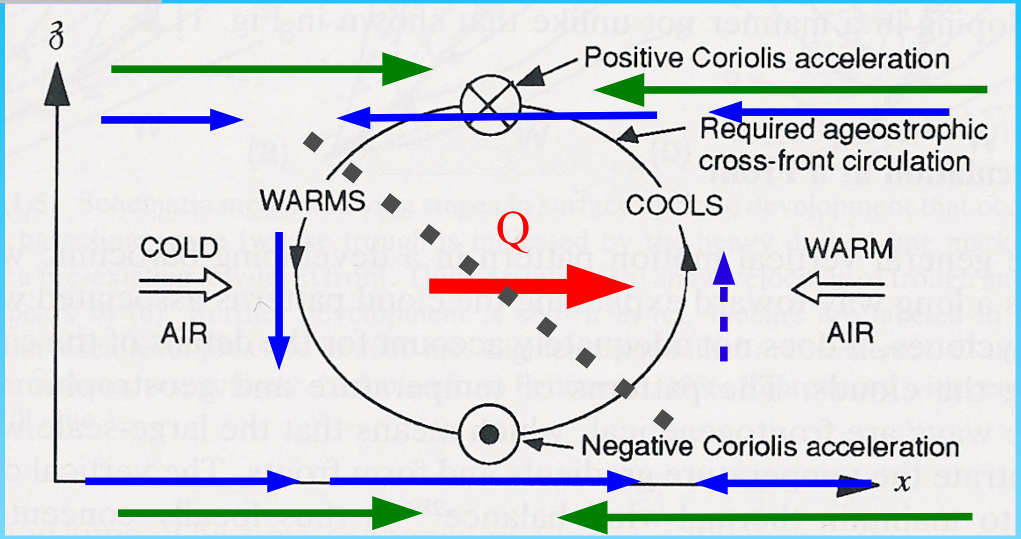
$$2\nabla_H . \vec{\boldsymbol{Q}} = N^2 \nabla_H^2 w + f_0^2 \partial_{zz}^2 w$$

Convergence of $\vec{\boldsymbol{Q}}$ = max of w positive

Divergence of $\vec{\boldsymbol{Q}}$ = min of w negative

Case of frontogenesis by confluence

jet v_g  (entering in the figure)



$$Q_1 = -\partial_x u_g \partial_x b - \partial_x v_g \partial_y b$$
$$Q_2 = -\partial_y u_g \partial_x b - \partial_y v_g \partial_y b$$

Generation of the ageostrophic circulation

$$2\vec{Q} = N^2 \nabla_H w - f^2 \partial_z \vec{u}_a$$

ou in other words (omega equation)

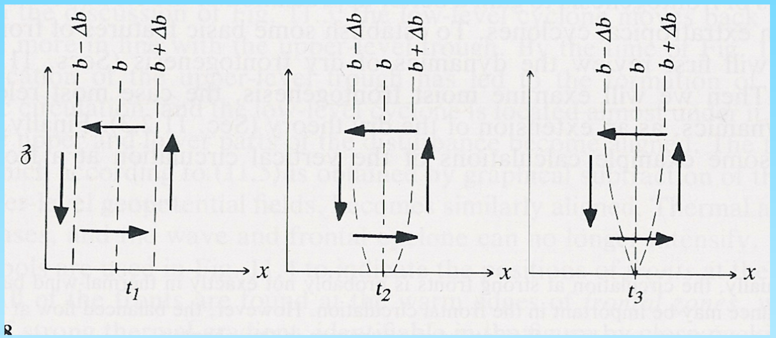
$$2\nabla_H . \vec{Q} = N^2 \nabla_H^2 w + f^2 \partial_{zz}^2 w$$

Convergence of $\vec{Q}$ = max of w positive

Divergence of $\vec{Q}$ = min of w negative

Geostrophic circulation, ageostrophic circulation and total circulation in the transverse plane to the jet in the confluent case.

The ageostrophic circulaton would tend to produce a tilted front but cannot do it since it is excluded from the advecting flow upon the quasi-geostrophic approximation.



Limitation of the quasi-geostrophic approximation : the front is only formed near the ground where $w=0$ and not inside where the ageostrophic circulation counterbalances Q_1. Any horizontal oscillation of size L is damped over a depth $f L/N$ in the vertical.

Summary (1):

- The non divergent quasi-geostrophic flow satisfies the thermal wind balance but the advection by the geostrophic wind tends to destroy this balance.

- The balance is restored by the divergent ageostrophic wind which can be determined from the quasi-geostrophic wind. This slave ageostrophic component is part of the quasi-geostrophic model (with the limitation that its advection is neglected).

Other « free » components of ageostrophic may be superimposed to the « slave » component such as those associated with the gravity waves excluded from the quasi-geostrophic model (defined as a first order approximation in the Rossby number).

Summary (2):

- The vector **Q** allows to visualize the slave ageostrophic circulation

.

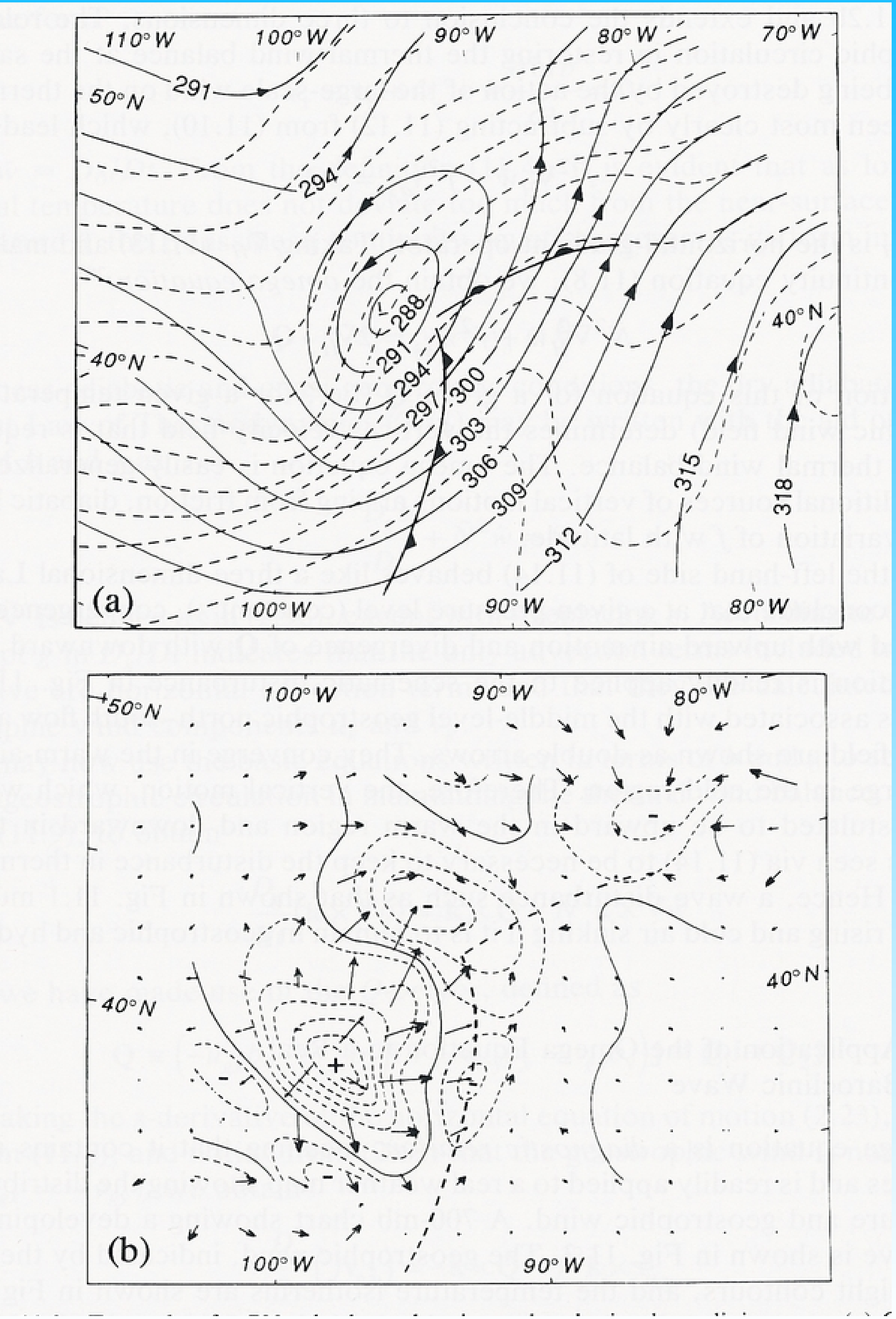
-In the case of a rectininear jet, the quantity -Q is proportional to the (mofified) Laplacian of the ageostrophic circulation streamfunction in the transverse plane.

- The horizontal divergence of **Q** is proportional to the (modified) Laplacian of the vertical component of the ageostrophic velocity: convergence of **Q** = ascending flow, divergence of **Q** = subsident flow.

-The geostrophic circulation can reinforce the temperature gradients (frontogenesis) by confluence or shearing of temperature lines. If advection by the ageostrophic flow is neglected, this intensification is limited to the upper and lower boundaries of the domain (surface and tropopause in the Eady model). Taking into account the ageostrophic advection allows to reinforce and tilt the inside part of the front.

Typical development of a baroclinic perturbation

Geopotential (solid) and temperature (dash) map at 700 hPa



Vecteur **Q** and divergence of **Q** (notice the convergence on the north and east sides of the depression and (warm front) and that on the south-east (cold front)

We see here that the vector **Q** visualizes the ascending and descending branches of the baroclinic circulation through the temperature lines.
The ascending motion tends to occur at smaller scale than the descending motion, near the developping frontal zones.

Frontogenesis

- Boussinesq equations
- Quasi-geostrophic frontogenesis
- Semi-geostrophic frontogenesis
- Influence of moisture
- Large-scale front
- Frontogenesis and storms

Semi-geostrophic frontogenesis

FRONTAL DYNAMICS SCALING (2D version transverse to the jet)

We assume that the length scale of the front is much larger than its transverse scale

$$L_y \gg L_x \text{ and } V \gg U$$

We assume that the Lagrangian derivative depends mostly of the cross front variations

$$D_t \sim \frac{U}{L_x}$$

Typically : $L_y \approx 1000$ km, $L_x \approx 100$ km, $U \approx 1$ m/s, $V \approx 10$ m/s

and we take $f_0 = 10^{-4}$ s^{-1}

Hence the ratios between inertial terms and Coriolis are

$$\frac{D_t u}{f_0 v} = \frac{U^2}{f_0 L_x V} \approx 0.01 \quad \text{et} \quad \frac{D_t v}{f_0 u} = \frac{V}{f L_x} \approx 1$$

Basic approximation: The along front component of the wind v is well approximated by its quasi-geostrophic component but the transverse component u is not. In addition, we neglect the variations of the wind along y, hence $\frac{\partial u}{\partial y} = 0.$

Thus we write $u = u_g + u_a$ where the two components are of the same order, and $v = v_g.$
and we take into account the ageostrophic terms in the advection

$$D_t = \partial_t + u\,\partial_x + v_g\,\partial_y + w\,\partial_z$$

The equations are

$$D_t v_g + f_0 u_a = 0 \quad \text{and} \quad D_t b + N^2 w = 0$$

$$\partial_x u_a + \partial_z w = 0$$

FRONTAL DYNAMIC EQUATIONS
(geostrophic moment equations)

We take $u = u_g + u_a$ where the two components are of the same order, and $v = v_g$.
and we take into account the ageostrophic terms in the advection

$$D_t = \partial_t + u\,\partial_x + v_g\,\partial_y + w\,\partial_z$$

The equations are thus

$$D_t v_g + f_0 u_a = 0 \quad \text{et} \quad D_t b + N^2 w = 0$$
$$\partial_x u_a + \partial_z w = 0$$

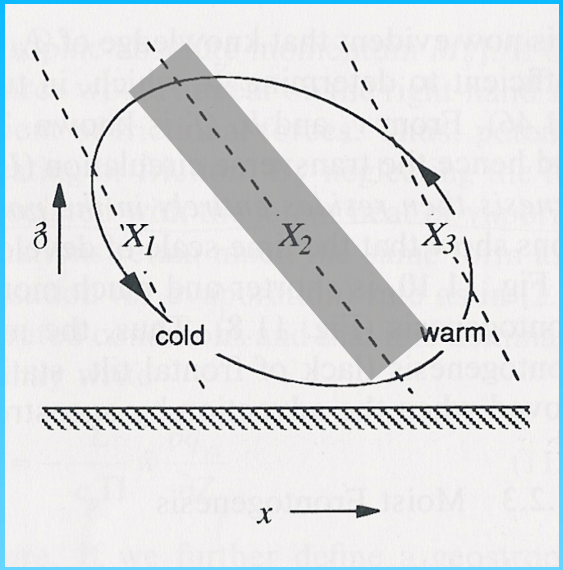
The time derivatives of the two terms of the thermal gradient are extracted

$$f_0 D_t \partial_z v_g = -Q_1 - \underline{\partial_z w\ \partial_x b} - f_0^2 \partial_z u_a - \underline{f_0 \partial_z u_a \partial_x v_g}$$
$$D_t \partial_x b = Q_1 - (N^2 + \underline{\partial_z b})\partial_x w - \underline{\partial_x u_a \partial_x b}$$

We see new terms (underlines) involving the vertical derivatives of the ageo-strophic circulation which reinforce the temperature gradient
After combining and rearrangement, we obtain the Sawyer-Eliassen equation:

$$N_s^2 \partial_{xx}^2 \psi - 2 S^2 \partial_{xz}^2 \psi + F^2 \partial_{zz}^2 \psi = -2 Q_1,$$

with $N_s^2 = N^2 + \partial_z b$, $F^2 = f_0 (f_0 + \partial_x v_g)$ and $S^2 = \partial_x b$.
This equation is elliptic and can be solved if $F^2 N_s^2 - S^4 > 0$, that is if the potential vorticity is positive (see problem)



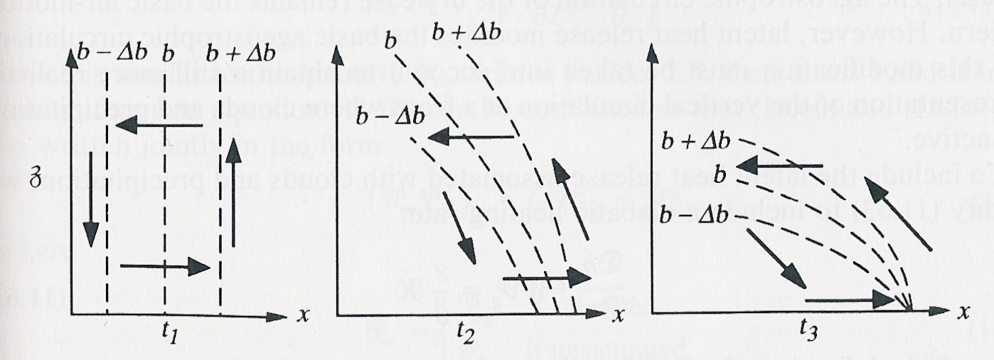
In gray, location of the front

The effects of advection by the ageostrophic equation are

- the formation of a tilted front towards the cold region
- the reinforcement of the wind in the cyclone to the expense of the anticyclone (see below)
- the shrinking of the area of the warm surface anomaly and the expansion of the area of the cold anomaly
- the inverse effect at the top boundary

Frontogenesis is forced by the large-scale flow but is also reinforced by the convergence of the ageostrophic circulation which increases with the reinforcement of the flow. In non dissipative situation, an infinite value of vorticity may appear in a finite time on the front.

Development of a ground front
It is easier to get a front at the ground because w cannot counterbalance the intensification of temperature gradient.



SEMI-GEOSTROPHIC EQUATIONS
(abridged form)

We define $X = x + \frac{1}{f} v_g$, such that $D_t X = u_g$,

and we change the coordinates (x, y, z, t) into $(X, Y = y, Z = z, \tau = t)$

This leads to $\partial_x = (1 + \frac{\partial_x v_g}{f}) \partial_X$

$\partial_z = \partial_Z + \frac{\partial_x b}{f^2} \partial_X$ and $\partial_y = \partial_Y + \frac{\partial_y v_g}{f} \partial_X$

thus $(1 + \frac{\partial_x v_g}{f})(1 - \frac{\partial_X v_g}{f}) = 1$

If $\Phi = \phi + \frac{1}{2} v_g^2$, hence $\partial_X \Phi = \partial_x \phi = f\, v_g$

$\partial_Y \Phi = \partial_y \phi = -f\, u_g$ and $\partial_Z \Phi = \partial_z \phi = b$

The potential vorticity is $q = \vec{\omega} \vec{\nabla} (b + \bar{b})$ with $\vec{\omega} = (-\partial_z v_g, 0, f + \partial_x v_g)$.

It is conserved $(D_t q = 0)$ and we have

$$q = \partial_Z (b + \bar{b})(f + \partial_x v_g) = f \frac{\partial^2_{Z^2}(\bar{\Phi} + \Phi)}{1 - \frac{1}{f^2} \partial^2_{X^2} \Phi}$$

or approximately

$$\frac{q}{f} \approx (N^2 + \partial^2_{Z^2} \Phi)(1 + \frac{1}{f^2} \partial^2_{X^2} \Phi) = N^2 + \partial^2_{Z^2} \Phi + \frac{N^2}{f^2} \partial^2_{X^2} \Phi$$

Under this approximation,

the quantity $\partial^2_{Z^2} \Phi + \frac{N^2}{f^2} \partial^2_{X^2} \Phi$

is conserved by the geostrophic flow in the space (X, Y, Z) which is such that

$D_t X = u_g = -\frac{1}{f} \partial_Y \Phi$ et $D_t Y \approx v_g = \frac{1}{f} \partial_X \Phi$

In particular, we have

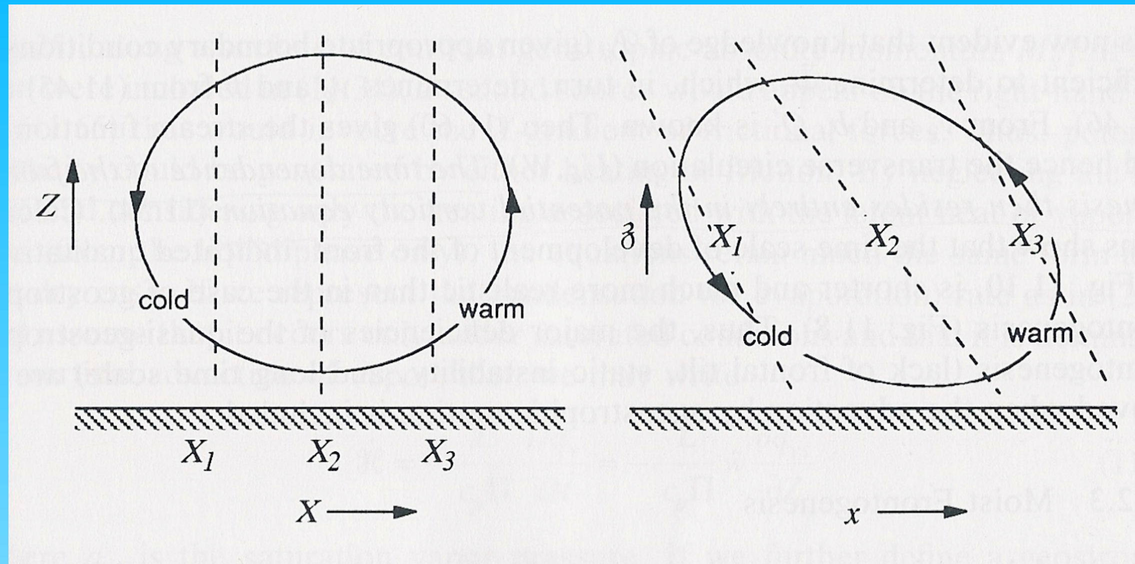
$$-2 Q'_1 = \partial_X (\frac{q}{f} \partial_X \Psi) + f^2 \partial^2_{Z^2} \Psi$$

with $Q'_1 = -\partial_X u_g \partial_X b - \partial_X v_g \partial_Y b$

and $U_a = \partial_Z \Psi$, $\quad W = \partial_X \Psi$

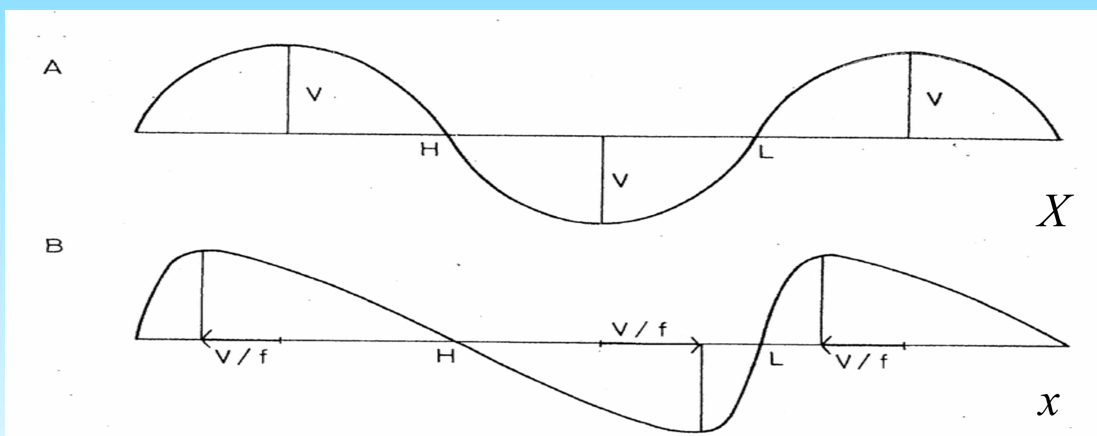
the ageostrophic circulation in the transformed frame.

The circulation in the original coordinate is obtained by transformation of the circulation quasi-geostrophic from the coordinates (X, Y, Z).



Tilt of the surfaces $X : \partial_z X = \frac{1}{f} \partial_z v_g$

Maximum tilt where the vertical shear ix maximum



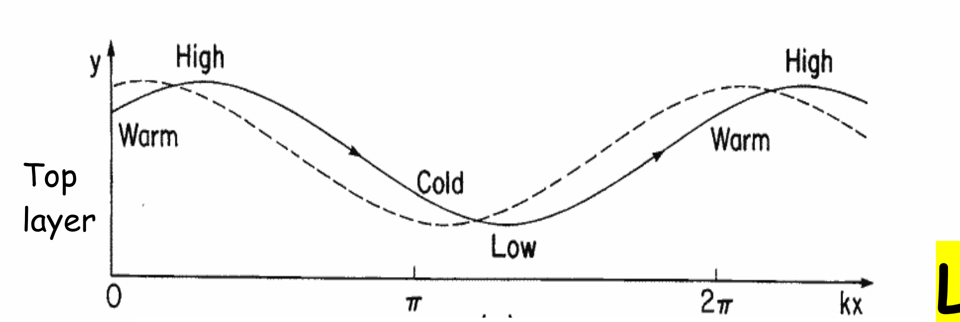
$Vg(X)$

$Vg(x)$

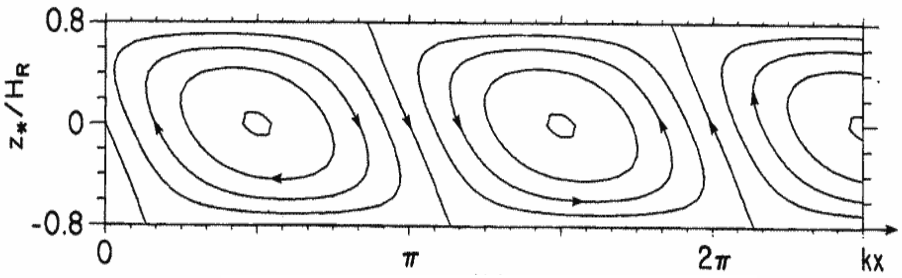
$\partial_x X = 1 + \frac{1}{f} \partial_x v_g$ shows that, in the real space, the isolines of X, and hence of v_g, get closer inside a cyclone ($\partial_x v_g > 0 \rightarrow \partial_x X > 1$) and get separated inside an anticyclone

Linear Eady model

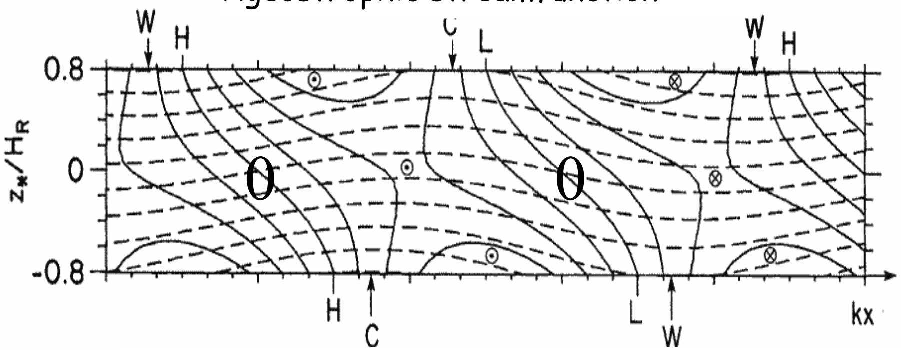
-> development of a cold front but not a warm front



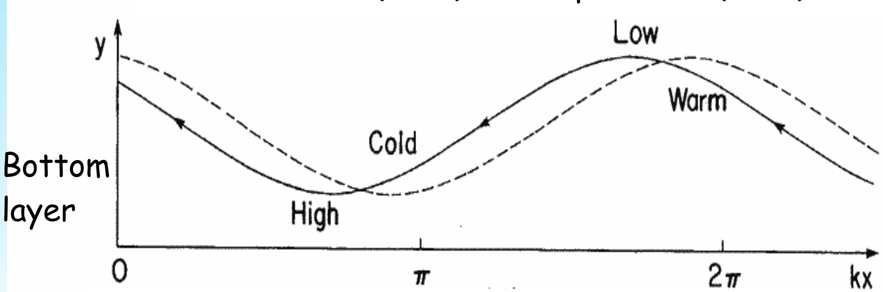
Geopotential (solid) and temperature (dash)



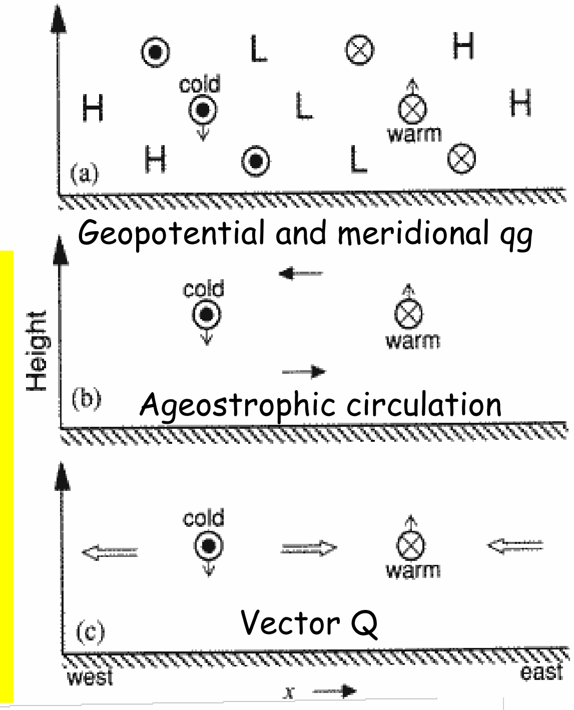
Ageostrophic streamfunction

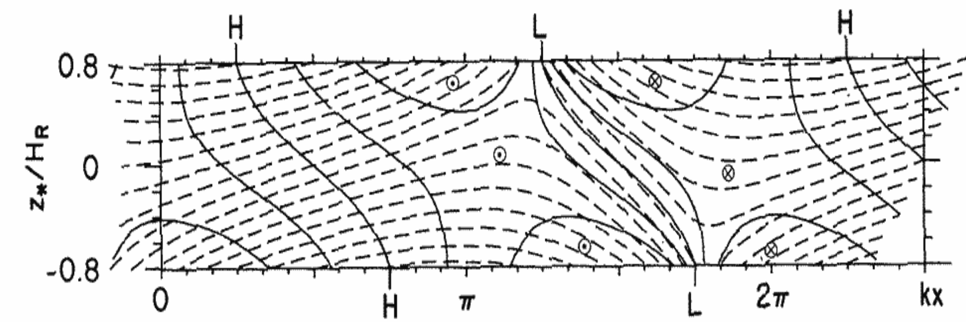


Meridional wind (solid) et temperature (dash)

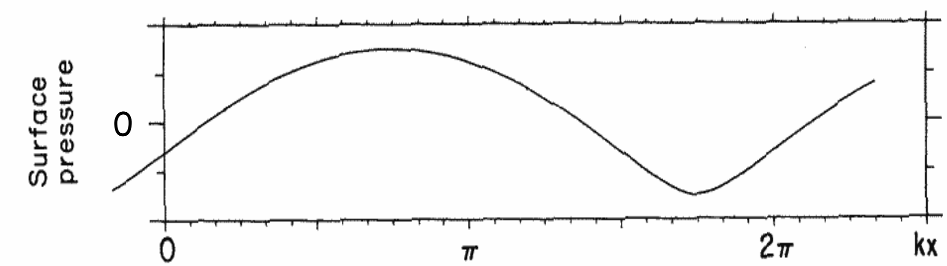


Geopotential (solid) and temperature(dash)





Meridional wind (solid) et temperature (dash)

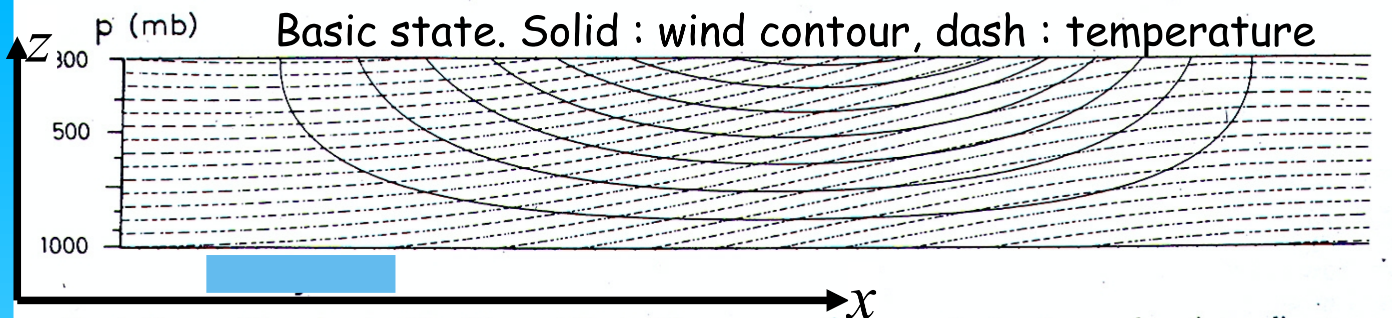


Surface meridional wind

Quasi-geostrophic

Semi-geostrophic

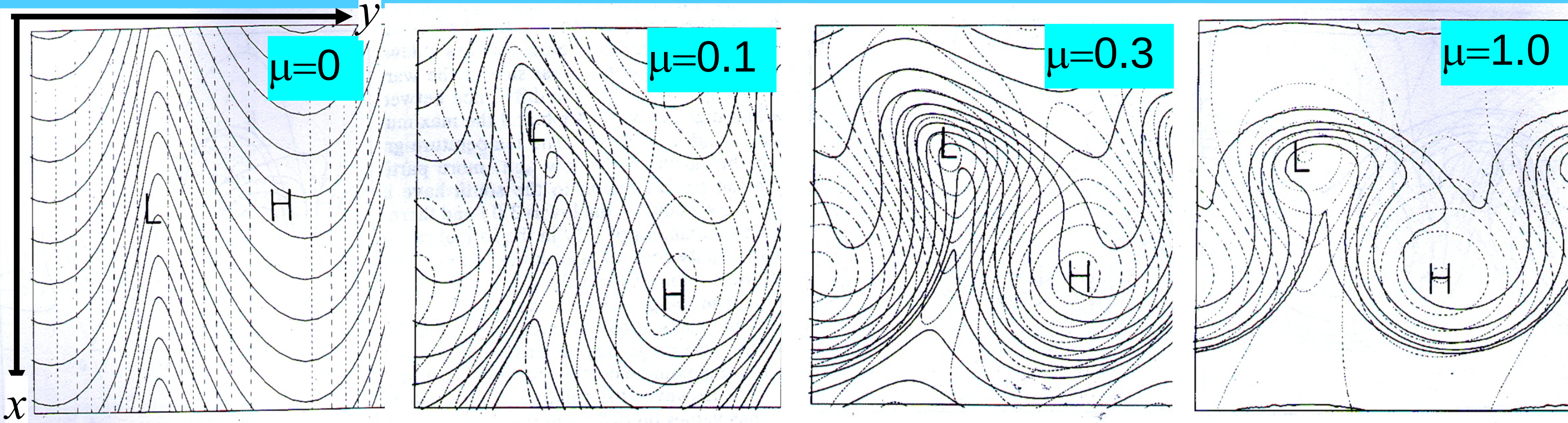
Frontogenesis: role of the horizontal shear
(towards more realism in the eady model)



Hoskins et West, 1979, JAS

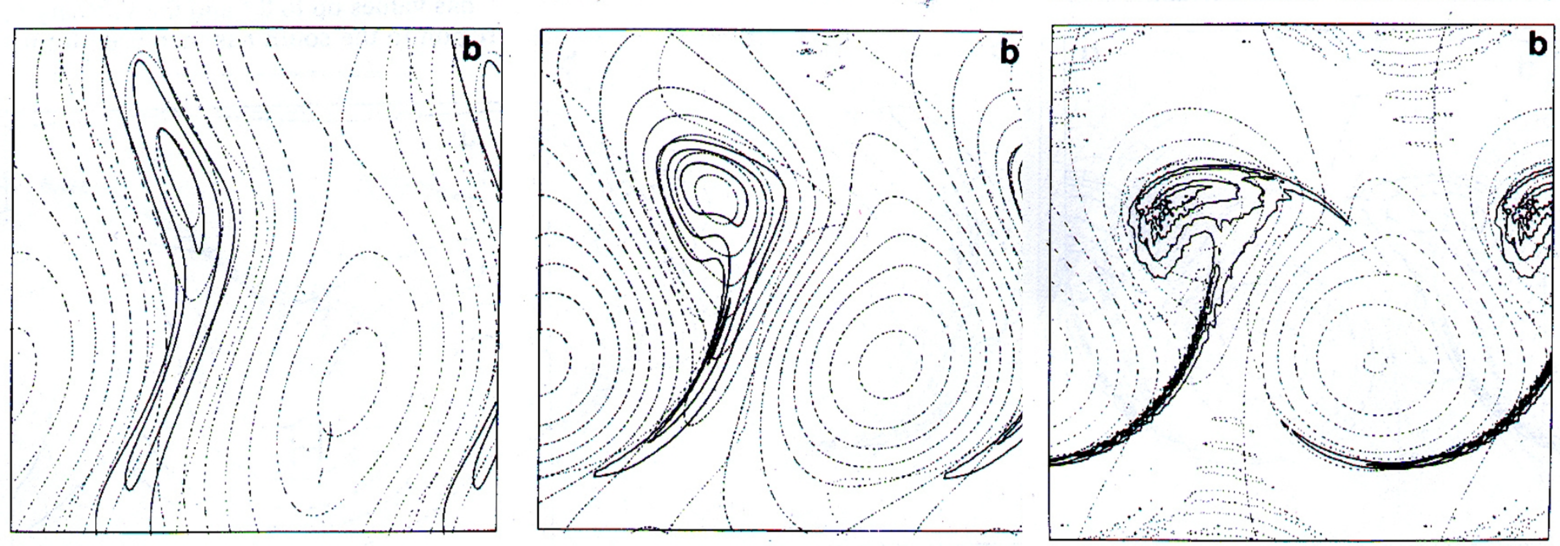
$$V = z - \frac{\mu}{2}\left(z + \frac{\sinh l\, z}{\sinh l}\cos l\, x\right)$$

$\mu = 0$: *flow without horizontal shear;* $\mu = 1$: *full jet, as shown above*



Top row :
geopotential (dash) and temperature (solid) at the ground

Bottom row:
geopotential (dash) and vorticity (solid) at the top **22**

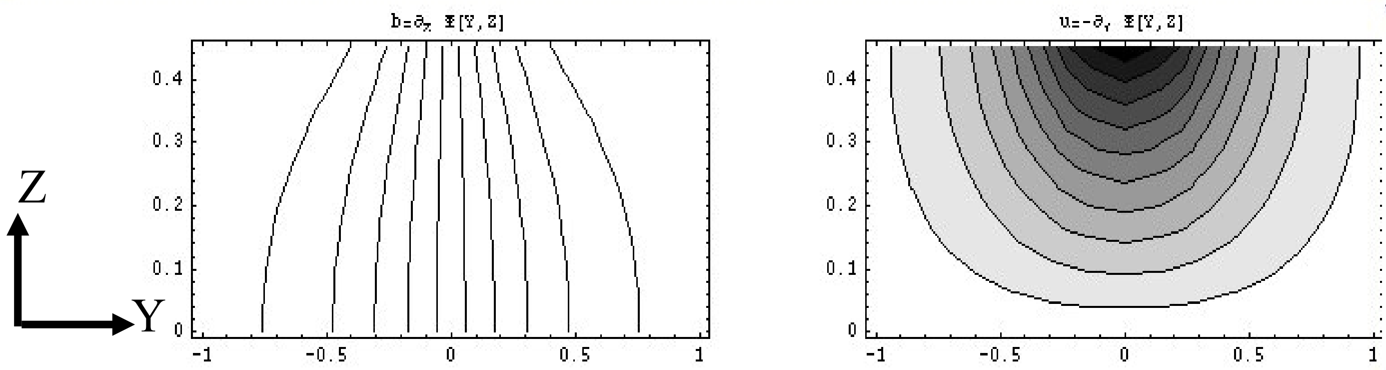


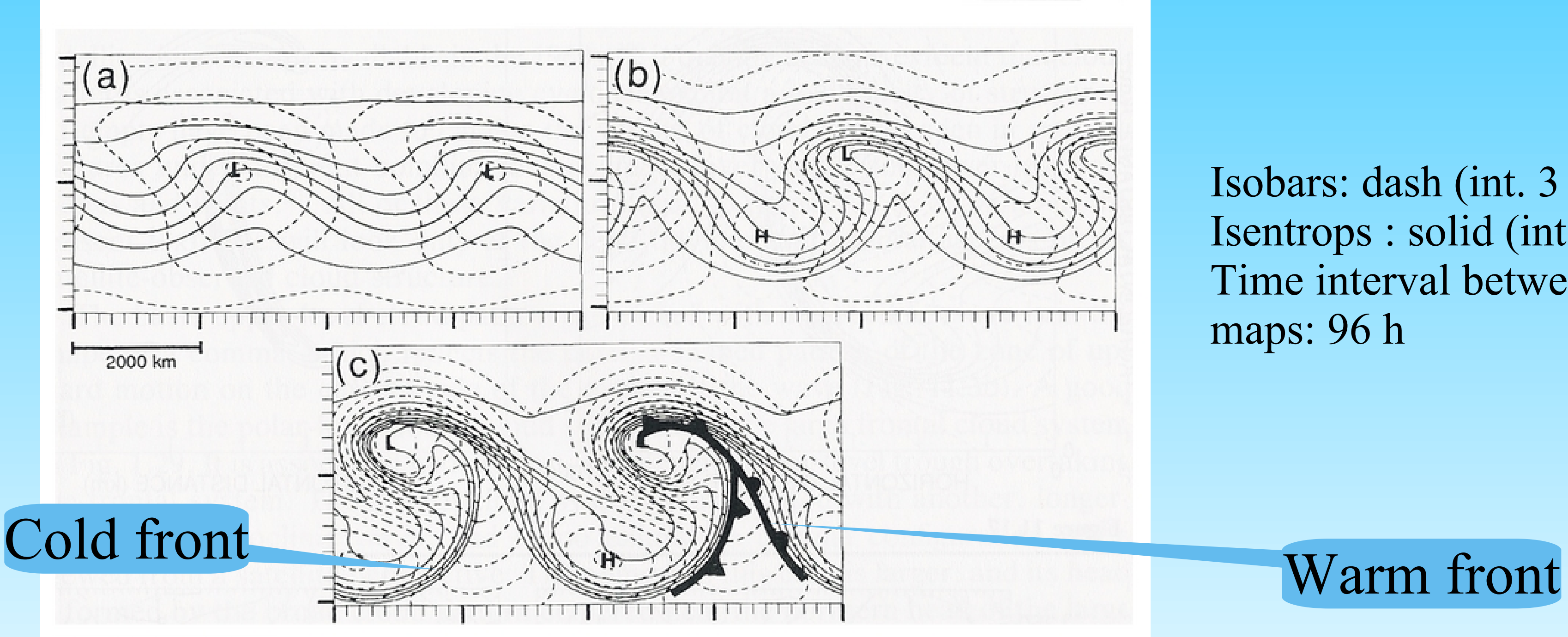
Frontogenesis: towards more realism with a better basic flow

Davies, Schar, Wernli, 1991, JAS

$$\Phi[Y\_, Z\_] := \frac{1}{2}\left(\text{ArcTan}\left[\frac{Y}{1+Z}\right] - \text{ArcTan}\left[\frac{Y}{1-Z}\right]\right) - 0.12\, Y\, Z$$

```
Show[GraphicsArray[{g1, g2}]]
```

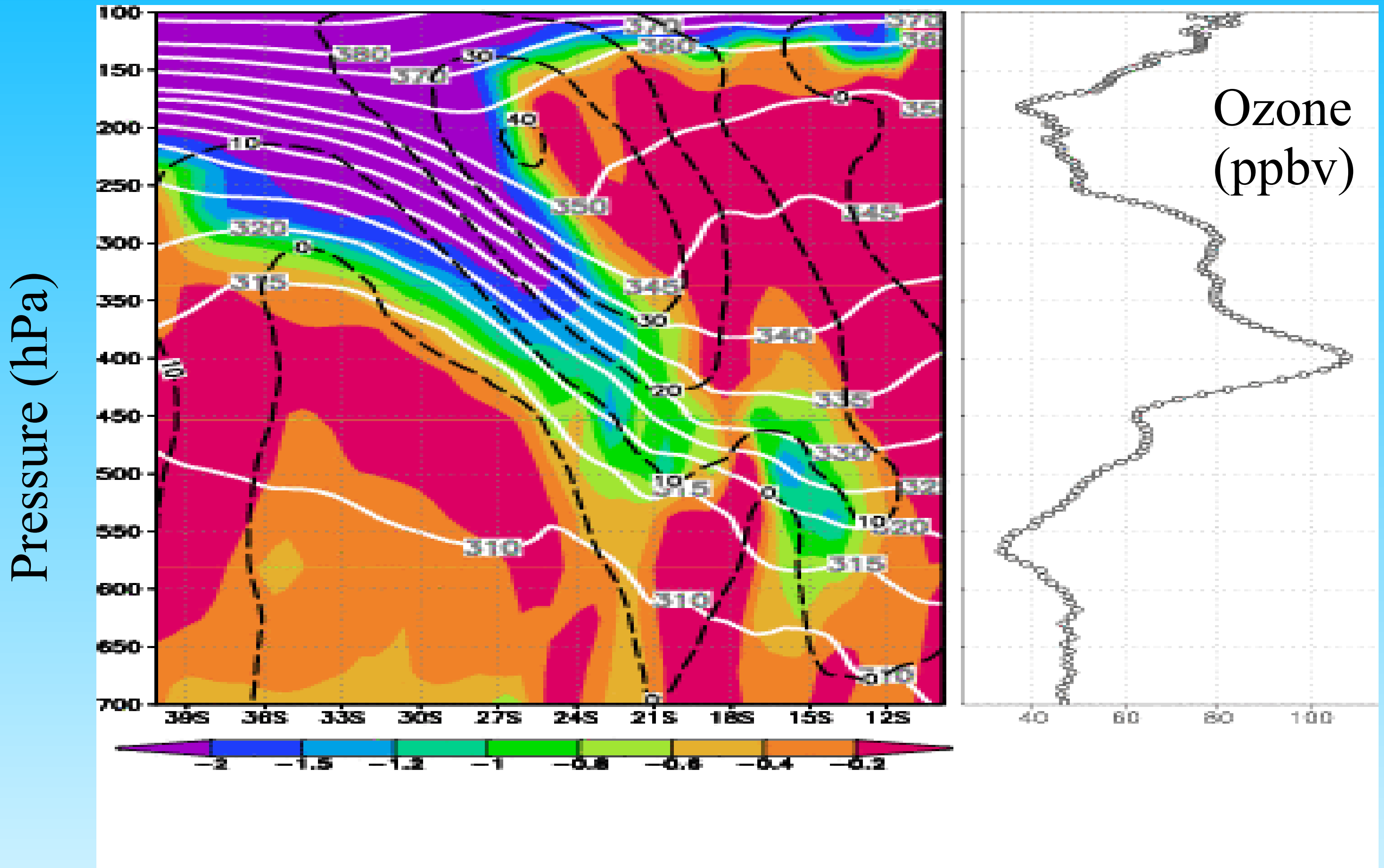




Isobars: dash (int. 3 hPa);
Isentrops : solid (int. 2K)
Time interval between two maps: 96 h

Development of a « realistic » baroclinic perturbation under the semi-geostrophic assumption.

Upper level frontogenesis and tropopause foliation

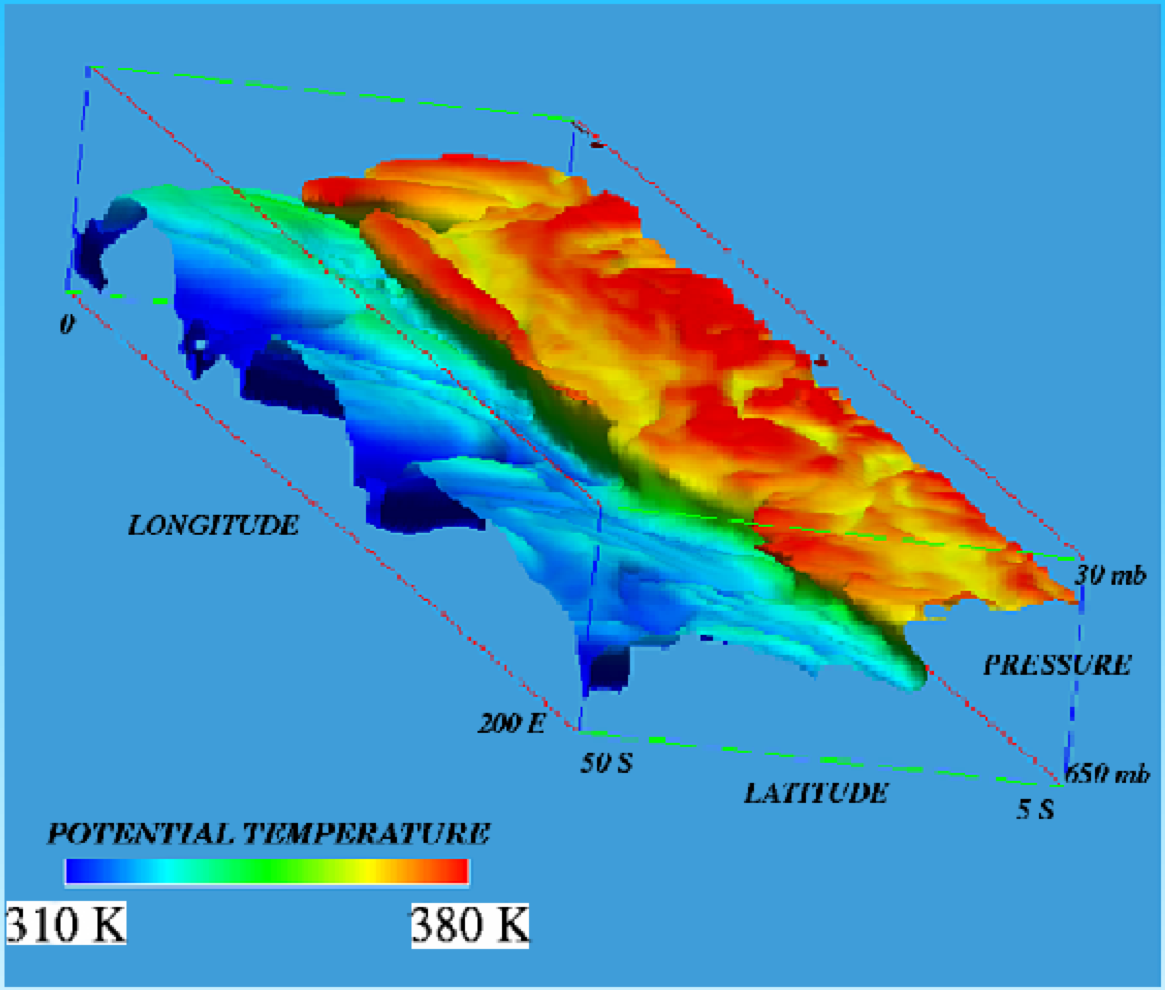


Upper level frontogenesis at tropopause. Generation of a tropopause fold and injection of stratospheric air in the troposphere. Baray et al., GRL, 2000

Frontogenesis

- Boussinesq equations
- Quasi-geostrophic frontogenesis
- Semi-geostrophic frontogenesis
- Influence of moisture
- Large-scale front
- Frontogenesis and storms

Frontogenèse d'altitude et foliation de tropopause (2)



Tropopause fold on a rectilinear jet over 200 degrees of longitude.
Baray et al., GRL, 2000