Attachment line swept wing "instability": a validation of local analysis

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Goals

We perform a global receptivity and sensitivity analysis of a swept-wing, incompressible boundary layer in a domain covering the attachment line as well as an extended region downstream of it. Despite performing our analysis in a stable flow regime — all our eigenvectors are decaying — we provide qualitative connections with previous local and global results in unstable regimes: the identification of the most receptive and sensitive regions within our domain provides an explanation for the validity of the local analysis results. The tools used are the one of modal analysis — eigenvectors and eigenvalues decomposition — and optimization theory. Receptivity and sensitivity form the foundation for the passive and active manipulation of the flow by applying control-theoretic means.

Model

$$\mathcal{R}(\mathbf{Q}) \equiv \begin{cases} \partial_t \mathbf{U} + \nabla\mathbf{U}\,\mathbf{U} - \nu\Delta\mathbf{U} + \nabla P & = \mathbf{F} \\ \nabla\cdot\mathbf{U} & = 0 \end{cases}$$

The boundary conditions used are:

- *inflow*: velocity and pressure are given from the inviscid solution
- *solid boundary* : velocity is zero and the equation for the pressure is applied with a modified stencil
- *outflow* : pressure is given from the inviscid solution and the momentum equations are applied with a modified stencil

Open Questions

- Pressure boundary conditions for the incompressible Navier Stokes equations
- How does multigrid behaves for complex numbers?
- Is it possible to compute eigenvalues and eigenvectors using the same multigrid structure?

References

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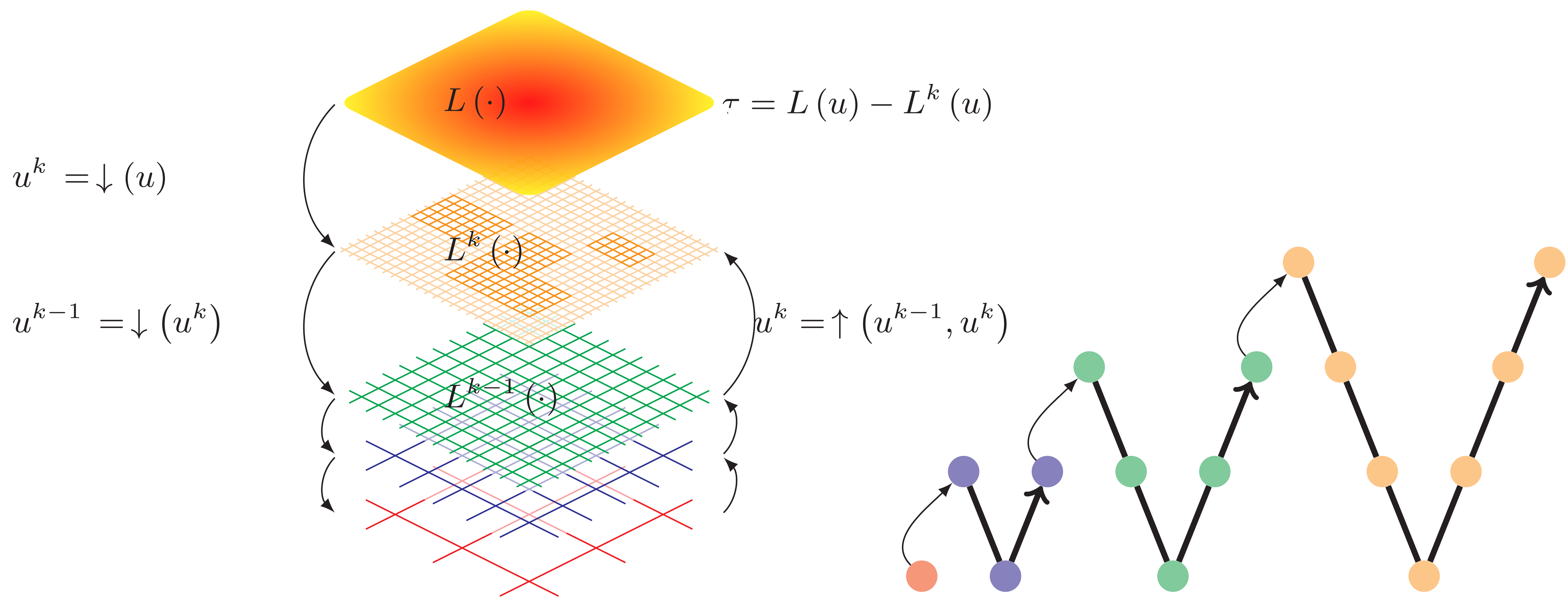
Multigrid

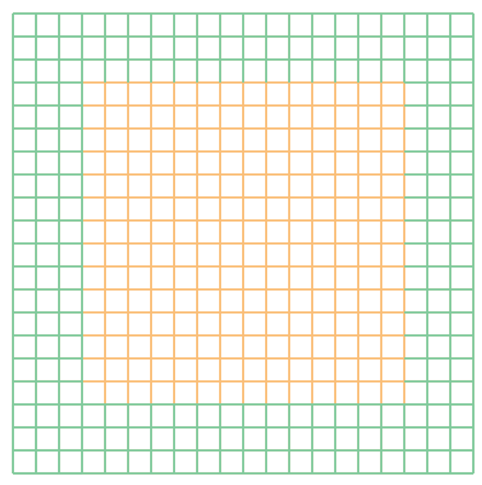


The FAS (Full Approximation Storage) multigrid scheme is used to solve the nonlinear, steady-state Navier Stokes Problem. The FAS algorithm has two main advantages with respect to the more widely known CS (Correction Scheme): it can address the non-linear problem without using an external Newton iterator and provides an efficient and easy way to obtain a grid refinenement only where necessary. On each grid level the linearization of the continuous problem is discretized and written in the form:

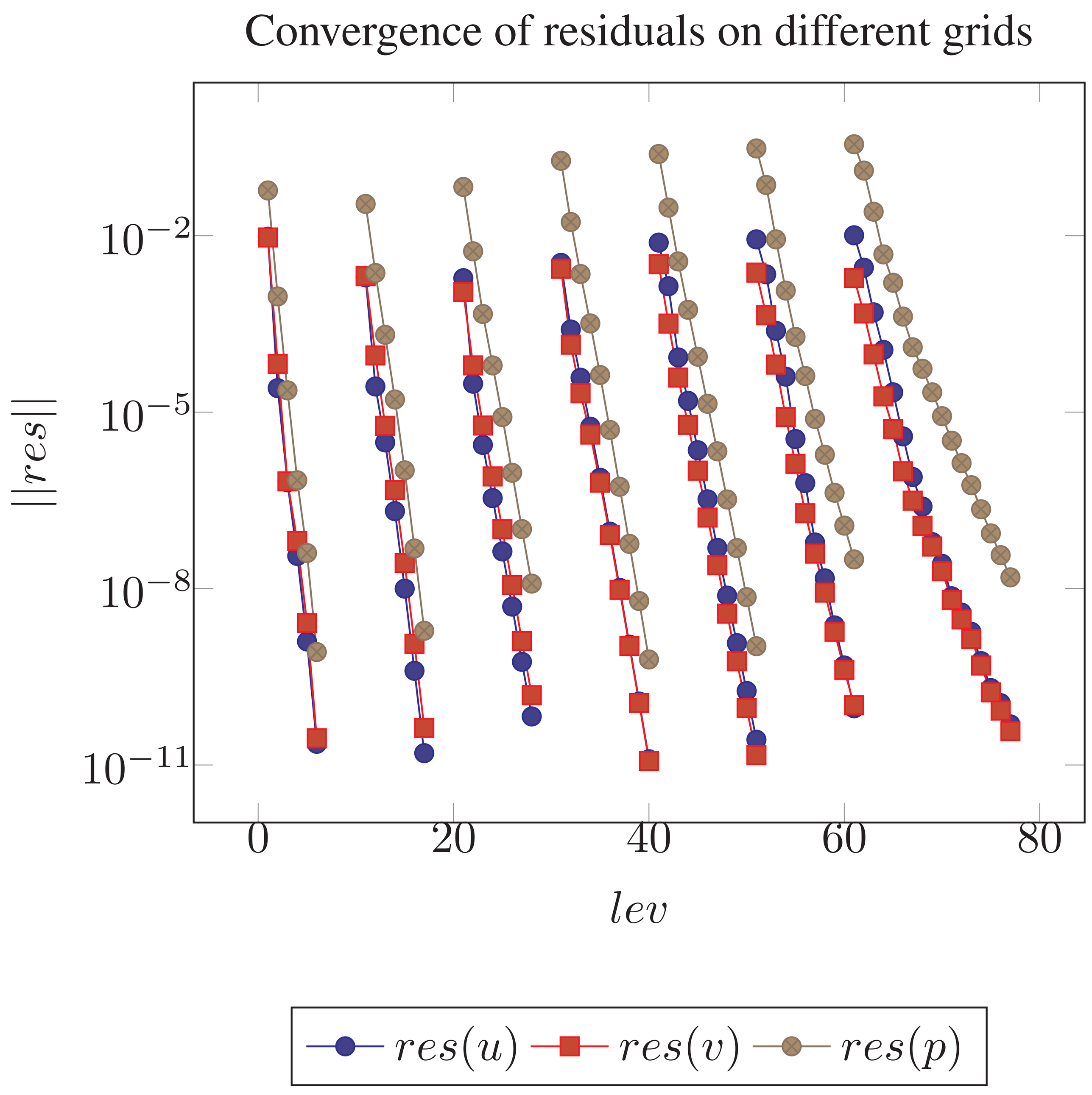
$$\frac{\partial L^k}{\partial u}\delta u^k = \tau^k_{k+1}$$

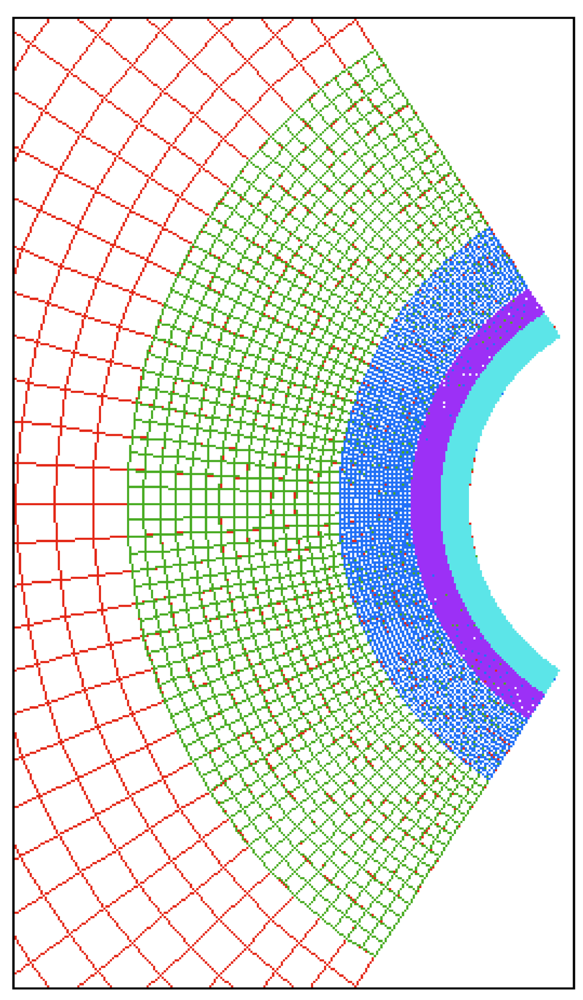
where $\tau^k_{k+1} = L^k\left(I^k_{k+1}u^{k+1}\right) - I^k_{k+1}\left(L^{k+1}u^{k+1}\right)$ is the "*fine to coarse defect correction, a correction to the coarse-grid equation designed to make its solution coincide with the fine grid solution*" [5].

On every level except the coarsest, the domain is decomposed in two different, possibly overlapping, subdomains. In the interior subdomain (orange) the equations are relaxed one by one with some (2 or 3) block-line Gauss Seidel sweeps performed in a downstream direction. The boundary subdomain (green) is instead solved [6].



Results





The convergence history of the residuals as a function of the number of V-cycles per level are shown for different grids. A detail of the ensemble of grids used is shown on the right. Each grid is obtained by applying a conformal mapping to a rectangular, equispaced grid. A continuation method is used in order to guarantee the convergence of the algorithm: the Re number is increased proportionally to the mesh size of the finest grid used in each moment (i.e. $Re \sim dh^{-1}$). The first series of data — when only the red grid is used — corresponds to $Re = 80$ and the last is for $Re = 5120$.

The number of iterations required to solve the equations increase with the Re and/or the inverse of the mesh size. The situation gets worse when a bigger downstream part of the geometry under consideration (in this case, a circle) is used.

A possible origin of this performance degradation is to be searched in the downstream relaxation process, which is at the moment performed along rays from the inflow to the solid boundary. While the velocity field is somehow aligned with the rays in the upstream part of the domain, this is no more true close to the outflow boundary, where the residual convergence is seen to be slower. A solution could be to change the direction of the sweep in this area.